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A paper to be presented at the 15th General Meeting of the American Institute of Electrical Engineers, Omaha, June 27th, 1898.

[ADVANCE COPY—SUBJECT TO REVISION.]

DIELECTRIC STRENGTH OF AIR.

BY CHARLES PROTEUS STEINMETZ.

To investigate the dielectric strength of air, tests were made of the striking distances in air between parallel cylinders of different diameters, between sharp points, and between spheres of different sizes. Further tests were made on the effect of inserting a conductor in or near the path of the disruptive discharge and investigating its influence on the striking distance.

As source of power, an A-8 smooth core alternator of the Thomson-Houston type was used, of about 30-K. W. capacity and a frequency of 125 cycles, giving practically a sine wave. In other tests a 60-K. W. high-frequency ironclad alternator of the Thomson Houston type was used, giving the well known saw-tooth form of wave. The alternator was driven by a direct current motor of ample capacity.

With alternating E. M. F.'s of only 125 cycles, the luminous phenomena, as brush discharges from conductors, etc., were much less marked and brilliant than observed even at lower voltages with oscillating currents of very high frequency, as given by a Thomson transformer.

For the purpose of these tests four high-potential transformers were built, of the ratio of transformation 1 : 18 and of a rating of about 7.5 K. W. each. They are of standard air-blast type, but were used when immersed in large wooden tanks filled with oil.

Each of these transformers contains two primary and two secondary coils, the two high-potential secondary coils in the center, the low-potential primary coils at the outside.

The utmost care was taken in the insulation of these transformers and it proved very successful, since over and over again

the voltage has been run up to 150,000 volts and beyond and later even to 172,000 volts without ever breaking down the transformer.

The high-potential circuits of the four transformers were connected in series. To avoid the danger of a disruptive discharge taking place from the outside of the high-potential transformer at one end of the series through the low-potential coils to the outside of the high-potential transformer at the other end of the series, the low-potential primaries were not connected directly to the generator in parallel with each other, but were connected to the high-potential sides of four standard type F transformers of

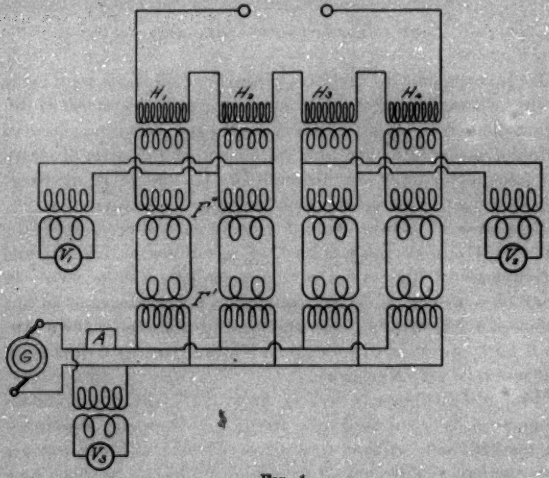

FIG. 1.

General Electric make of 7.5 K. W. each, and these transformers were again fed by another set of four standard type F 7.5 K. W. transformers, the latter being connected in parallel to the generator.

Thus the diagram of connections was as follows (Fig 1): The generator G feeds in parallel the high-potential sides of four transformers F' which step down in the ratio 10 : 1 or 10 : .5. These transformers F' feed four transformers F'' which step up in the ratio 1 : 10 or .5 : 10 and feed with their high potential side the low-potential coils of the four transformers H_1, H_2, H_3, H_4.

An internal disruptive discharge thus has to pierce two high-potential transformers H_1 and H_4 and four standard F transformers.

Incidentally hereby the ratio from the generator to high-potential circuit could be changed between 1 to 360, 1 to 720, and 1 to 1440 by changing the ratio of the F transformers, that is connecting their low-potential coils in multiple or in series.

The generator was driven by a direct current motor and was separately excited. The voltage was varied by varying the generator excitation, by varying the ratio of transformation of the F transformers, and by cutting more or less of the high-potential transformers out of circuit. Thus in the lowest readings only one high-potential transformer was used, in the highest readings all four. When cutting out transformers always the first transformer H_4, then H_1, and last H_3 were cut out, leaving ultimately only H_2 in circuit.

The F transformers were generally for the higher voltage used with the ratio 1 : 10 in F′ and .5 : 10 in F″.

The high potential voltage was derived by the ratio of transformation, and for this purpose two Weston alternating voltmeters with 10 : 1 voltmeter transformers were inserted on the low-potential side of the transformers H_2 and H_3, as shown by V_1 and V_2 in Fig. 1. They were calibrated before and after each test. A third voltmeter was inserted into the generator circuit with voltmeter transformer, and is shown as V_3. Its readings were not used for deriving the high-potential voltage, since too many transformations were intermediate. Thus it was used merely as a check and for guidance in operating the generator field rheostat.

The tests were made in the usual manner by setting the electrodes for a certain distance, then closing the generator switch and gradually raising the generator excitation while following the rise of the voltmeter needle, until the discharge took place and the voltmeter dropped. Then the exciter switch was opened, the voltmeter reading recorded, and the electrodes reset for the next test.

For the tests between points, $2\frac{1}{2}''$ sewing needles were used and for every test a new pair of needles taken, since it was not considered sufficient to re-sharpen the needles after the discharge had taken place.

For the tests between cylinders and spheres it was not considered sufficient to have the electrodes polished, since no polish can give an absolutely perfect surface without any minute un-

evenness. To secure a perfect smoothness, a film of liquid quicksilver was used as electrode surface, that is polished brass cylinders and spheres were used, and were immersed in a solution of mercuronitrate and afterwards wiped off with a clean cloth. The fairly thick film of quicksilver deposited on the electrode surface forms a liquid and thus perfectly smooth surface. After every discharge the electrodes were reamalgamated.

The effect of this precaution was that even with very small distances, the discharge appeared constantly at the same voltage, while with merely polished electrodes with very small distances, a more or less erratic behavior takes place due to minute unevenness of the surface.

Before undertaking the final tests, a number of preliminary tests were made to determine,

1). The constants of the generator and the transformers and the total power thus available in the discharge circuit.

2). The equality of the ratio of transformation of all four sets of transformers, since only at two out of the four transformers was the voltage measured.

3). The constancy of the ratio of transformation, since it was relied upon for determining the high potential voltage.

4). The shape of the generator wave and its variation under the conditions of test.

1st.—The synchronous impedance of the generator was found as,

$$Z_0 = r_0 - j\,x_0 = 3 - 12j,$$

that is, at an excitation corresponding to the voltage v at open circuit with a current C in the armature, the E. M. F. $C\,r_0$ is consumed in phase with the current, the E. M. F. $C\,x_0$ in quadrature with the current, thus giving the terminal voltage $V = C\,(r_0 - j\,x_0)$.

Of the transformers F′ and F″, of rating F–125–7.5–1040–104–52, reduced to the high potential or 1000-volt coils, it is:

The total impedance,

$$Z^1 = r^1 - j\,x^1 = 2.1 - 20\,j.$$

The primary admittance, that is, admittance of the exciting circuit,

$$Y^1 = g^1 + j\,b^1 = (.13 + .24\,j)\,10^{-3}.$$

The total number of primary turns is 280, the total number of secondary turns 28, of which, as before said, two could be connected in parallel. The cross-section of the magnetic circuit is 32.4 sq. in.

Of the transformers H_1, H_2, H_3, H_4 of rating OC-125,-7.5,-27000,-1500 reduced to the low potential or 1500-volt coils, it is:

Total impedance,

$$Z_1 = r_1 - j\,x_1 = 2.5 - 8.0\,j.$$

The primary admittance,

$$Y_1 = g_1 + j\,b_1 = (.32 + .5j)\,10^{-3}.$$

The total number of high-potential turns is 3458 per transformer, of wire No. 24 B & S, subdivided in two coils.

The total number of low-potential (1500 volt) turns is 192, subdivided in two coils also. The cross-section of the magnetic circuit is 46.5 sq. in. in the center, 72.9 sq. in. in the outside shell.

Herefrom we get for the whole system in the connection used for the highest voltage, that is with F′ connected 10 : 1, and F″ connected .5 to 10, reduced to the high-potential discharge circuit.

Total impedance inclusive generator,

$$Z = 4 \times 18^2\,Z_1 + 4 \times 18^2\,Z^1 + 4 \times 36^2\,Z^1 + 144^2\,Z_0$$
$$= (80 - 390\,j)\,10^3 \text{ ohms},$$

or absolute:

$$z = 398 \times 10^3 \text{ ohms},$$

or reduced to the generator circuit:

$$Z_2 = r_2 - j\,x_2 = 3.9 - 18.8\,j \text{ ohms}.$$

Total admittance,

$$Y = \tfrac{1}{4}\,\tfrac{1}{18^2}\,Y_1 + \tfrac{1}{4}\,\tfrac{1}{18^2}\,Y^1\,\tfrac{1}{4}\,\tfrac{1}{36^2}\,Y^1$$
$$= (.37 + .62\,j)\,10^{-6} \text{ mhos}.$$

or reduced to the generator circuit,

$$Y_2 = (7.7 + 12.8\,j)\,10^{-3} \text{ mhos}.$$

Thus with 150,000 volts between the discharge terminals at the total ratio of transformation 144, the terminal voltage of the generator is about 1,100 volts and its field excitation corresponds to an induced voltage at open circuit of about V = 1,280 volts. It follows herefrom that the maximum current which can be produced in the discharge circuit is:

$$\frac{V}{144\sqrt{r_2^2 + x_2^2}} = .465 \text{ amps.},$$

and the maximum power available in the discharge circuit is:

$$\frac{V^2}{2\,(r_2 + \sqrt{r_2^2 + x_2^2})} = 35.5 \text{ K. W.}$$

At this voltage the maximum density in the core of the transformers H is 42.5 kilo lines per sq. in., or 6,600 lines per sq. cm. In the transformers H'' it is 41.5 kilo lines per sq. in., or 6,400 lines per sq. cm., hence far below saturation.

2nd.—To determine the equality of the ratio of transformation, all four high-potential transformers were connected in parallel with their high-potential coils and in series with their low-potential coils, as shown in Fig. 2, and the voltages across the low-potential coils read. They were,

a	b	c	d
419	408	401	401

showing that the first transformer has a slightly higher ratio of transformation, and the last two a slightly lower ratio than the average.

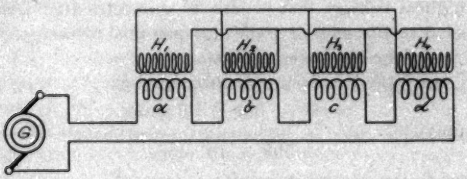

FIG. 2.

Allowance therefor was made in reducing the readings. The correction in the extreme case did not exceed 1.33%.

3rd.—Since the high potential was derived indirectly by voltmeter readings on the low-potential side of the transformers, it was necessary to determine whether the voltage read in this manner was correct and whether the capacity of the discharge circuit has any effect on the ratio of transformation.

For this purpose the high-potential transformers were connected in pairs in parallel with their low-tension and in series with their high-tension sides, the one pair used as step-up transformers, the other pair as step-down transformers, and the voltage read in both low-tension circuits, as shown in Fig. 3. Then step-up and step-down transformers were interchanged, that is, the generator was connected first at a, then at b. This test was repeated after

inserting an oil condenser into the high-potential circuit at *c*. This condenser consisted of twelve iron plates of about ½ sq. ft. each, separated by $\frac{2}{4}''$ of oil. Two such boxes of condensers were used in series, giving a total condenser surface of about 5½ sq. ft. per electrode and 1½″ oil distance, thus a capacity far in excess of that of any discharge circuit.

A number of tests were made, of which I give one set.

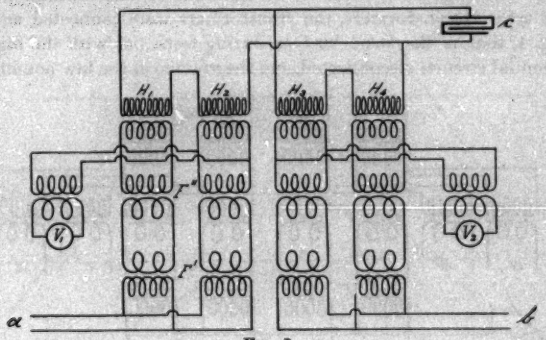

FIG. 8.

Generator at *a*: Ratio: 1 ÷ 360.				Generator at *b*:			
No Condenser.		Condenser.		No Condenser.		Condenser.	
V_1	V_2	V_1	V_2	V_1	V_2	V_1	V_2
11.5	11.0	11.0	11.0	10.1	11.8	10.6	11.9
25.2	23.5	20.3	19.0	24.2	25.1	19.8	20.6
44.0	42.8	30.0	28.5	43.2	45.0	29.5	30.5
66.8	64.2	39.9	38.3	65.3	67.0	39.4	40.4
93.0	89.2	45.6	43.7	91.0	92.7	44.5	45.8
102.1	98.5	56.0	53.5	100.4	102.0	54.7	56.0
117.6	113.0			115.8	117.3		
128.0	123.5			126.6	128.6		
134.4	129.6			132.2	133.6		

Average ratio:

1.0405	1.0455	1.0201	1.03

Average without condenser, 1.0303.

Average with condenser, 1.0396.

Hence 1.0303 is the change of ratio of transformation due to three times the exciting current. Thus the correction for the exciting current is 1.01%. The relatively very large condenser inserted into the high-potential circuit does not change the ratio of transformation by as much as 1%. Thus the very much smaller capacity of the discharge circuit is entirely negligible in its effect on the ratio of transformation.[1]

To investigate the equality of the ratio of transformation of the type F transformers, the transformers were connected as in Fig. 4, that is the same way as during tests, but with the high-potential circuits disconnected, and the voltage at the low-potential

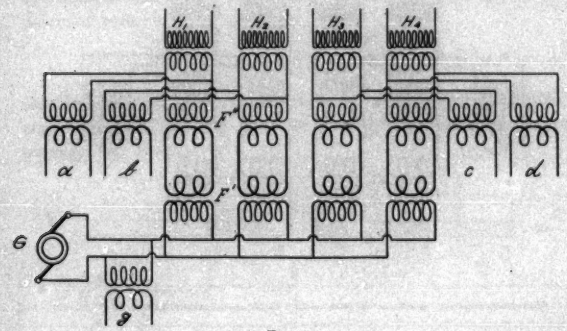

FIG. 4.

terminals of the high-potential transformers read at a, b, c, d and at the generator g. It was found:

g	a	b	c	d
50.3	47.5	48.1	47.9	47.8
74.6	70.0	70.7	70.6	71.0
100.6	97.0	97.2	97.4	97.3
197.3	129.8	123.2	123.0	123.0
352.8	337.3	339.2	338.2	339.1

Average $a, b, c, d = 338.45$,

1. This is due to the relatively very low self-induction of the step-up transformers compared with the self-induction of, for instance, a Ruhmkorff coil, etc.

Average ratio:

1.0425 | .9966 | 1.0022 | .9993 | 1.0019

that is, the four sets of intermediate transformers F′ and F″ are

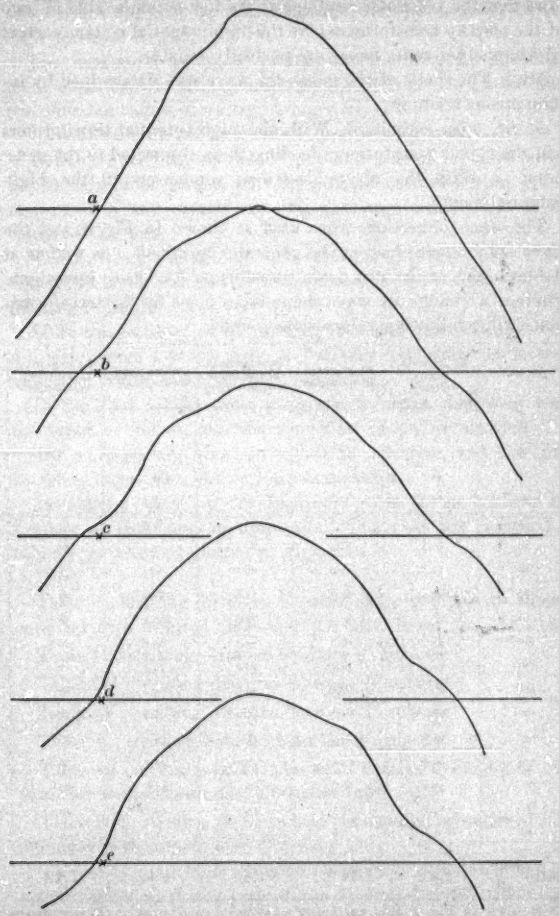

FIG. 5.

of identical ratio of transformation within a small fraction of one per cent., and the drop of voltage in them (which, however, is unessential for the test) amounts to 4.25%. Furthermore it follows that the voltmeter readings at the low-potential side of two of the step-up transformers give the high-potential voltage correct within one per cent.. hence are perfectly reliable.

4th.—The shape of the generator wave was determined by instantaneous readings,

a. At open circuit. *b*. With the high-potential transformers and the type F transformers feeding them connected to the generator. *c*. With the above described condensers in the high-potential circuit.

The same connections were used as shown in Fig. 3, and the wave shape determined at the generator terminals *a* as well as at the terminals of the step-down transformer *b*, so as to investigate whether a change of wave shape takes place by the transformation. The following values were found:

Degrees.	At generator, open circuit.	At generator, no condenser.	At step-down transformers, no condensers.	At generator, condenser.	At step-down transformer, condenser
10	13	7.25	5.8	5.3	3.7
20	98.5	18	14.4	26	25.8
30	44.1	35.5	29.7	43.3	42.3
40	64.4	52.6	48.2	56.2	53.9
50	13.6	61.6	56.5	64.8	60.7
60	82.5	66.0	60 1	72.6	69.3
70	89.6	71 9	66.5	79.9	77.5
80	95.5	77.3	72.8	85.0	81.9
90	97.7	80.1	74.2	86.1	82.3
100	95	77.5	72.5	83.6	79.8
110	91.4	74.25	70.8	79.2	75.1
120	86.5	70.6	66.6	74.6	70
130	76.8	61.6	58.6	69.4	65.8
140	63.2	49.9	48.1	59	56
150	50.5	41.25	40.2	42.1	38.7
160	36.6	28.6	27.1	23.6	21.8
170	19.9	13.3	13.4	14.75	13.4
180	2.6	.6	.9	7.6	8.3
Ratio $\frac{\text{max.}}{\text{eff}}$	1.42	1.44*	1.43	1.42	1.42

* As seen in Fig. 5, this particular wave has a small peak at the maximum reading which causes a slightly higher ratio, but is probably merely an error of observation of this individual reading.

that is 1.008 times the value corresponding to the sine wave.

Hence, the generator wave has within less than 1% the same ratio of maximum to effective value as the sine wave, and this ratio is not changed by transformation nor by capacity in the high potential circuit.

The maximum value of voltage can therefore be calculated from the effective value by multiplying by $\sqrt{2}$. In all the tests the effective values of voltage are given.

To conclude, we get as results of preliminary tests:

Voltmeter readings on the low-potential side of the high-potential transformers give a striking voltage correct within 1%, irrespective of the capacity of the discharge circuit, within the limits of the tests.

The wave of the smooth core alternator is practically a sine wave, and is not changed by transformation or by capacity of the discharge circuit.

Since the accuracy of the striking distance determination cannot well exceed 1 to 2%, these preliminary tests show the method as correct within the errors of observation.

In the tests all distances are given in inches, since they were measured in inches, and the reduction to cm. is omitted.

The voltages are given in kilo-volts effective, and the maximum voltages thus are $\sqrt{2}$ times as large.

In general each of the readings given in the table is the average of from two to four tests. The tests are recorded in tables I. to IX. and plotted in Figs. 6 to 17.

It is:

Table I., striking distances between sharp points with smooth core and with ironclad alternator, the latter in air, fog and steam.

Table II., striking distance between $\frac{1}{4}''$ spheres.

Table III., striking distance between $\frac{1}{2}''$ spheres.

Table IV., striking distance between $1''$ spheres.

Table V., striking distance between $2''$ spheres.

Table VI., striking distance between parallel cylinders of $.313''$ diameter with smooth core alternator only.

Table VII., striking distance between parallel cylinders of $.111''$ diameter with smooth core alternator only.

Table VIII., striking distance between $\frac{1}{4}''$ spheres with brass disks inserted into the path of the discharge, with smooth core alternator.

Table IX., striking distance between $\frac{1}{4}''$ spheres with wet plank parallel to the path of the discharge.

Fig. 6, sharp points with smooth core alternator, observations marked by crosses.

Fig. 7, sharp points with smooth core alternator. Average of two sets of tests. Observations marked by crosses.

Fig. 8, $\frac{1}{4}''$ spheres. Two sets of tests. Smooth core alternator.

Fig. 9, $\frac{1}{4}''$ spheres. Average of two sets of tests. Smooth core alternator.

Fig. 10, $\frac{1}{2}''$ spheres. Smooth core alternator.

Fig. 11, $1''$ spheres. Smooth core alternator.

Fig. 12, $2''$ spheres. Smooth core alternator.

Fig. 13, sharp points and spheres. Comparison of results with smooth core alternator. Sharp points showing dotted line.

Fig. 14, .313″ cylinders with smooth core alternator. Maximum electrostatic gradient marked by crosses.

Fig. 15, .111″ cylinders with smooth core alternator. Maximum electrostatic gradient marked by crosses.

Fig. 16, sharp points with ironclad alternator. Two set of tests in air, one in fog and one in steam. Smooth core alternator, curve marked by dash-dotted line.

Fig. 17, sharp points and spheres. Comparison of results with ironclad alternator. Curve of sharp points in dotted lines. Sharp points with smooth core alternator marked with dash-dotted lines.

The following has to be remarked regarding these tests:—

All distances given are net, that is, from the surface of sphere and cylinder to the surface of the other cylinder and sphere, etc. Connection of the high-potential transformers H_1, H_2, H_3, H_4 and the intermediary transformers F' and F'' with the generator G and the voltmeters V_1, V_2, V_3, were in all the tests, as shown in Fig. 1. As stated before, the voltmeter readings of V_1 and V_2 were averaged to calculate the striking voltage, while the voltmeter reading V_3 was used merely as check. An ammeter was also inserted into the generator circuit at A as check to observe capacity effects.

Generally the intermediary transformers near the generator, F', were connected for the ratio 1 to 10, the transformers F'' for the ratio 1 : 20.

For lower readings the high-potential transformers were gradually cut out in the order H_4, H_1, H_3, leaving ultimately only H_2 in circuit.

Under the floor of the testing room were a number of steam pipes from which some steam was escaping, so that the air in the testing room was always moderately moist.

TABLE I.

POINTS.

2¼″ needles. 125 cycles.

Smooth Core Alternator: A-10-30-1500.				Ironclad Alternator: A-10-60-1500.					
Distance, inches.	Kilovolts: effective.			Distance, inches.	Kilovolts: effective.				
d.	7-21-'96 * in air.	7-17-'96 † in air.	Average.	d.	7-24-'96 ‡ in air	Jan. '95 in air.	Average	Jan. '95 in fog.	Jan. '95 in steam
.25	4.25			.25	4 13				
.5	10.0			5	9.0	10 0	9.5	11.0	14.5
1.0	20.4			1.0	16 9	18.5	17.7	22.0	25.3
1.5	29.3			1.5	24.3	26.0	25.1	31.0	35.5
2.0	35.8			2.0	30.5	30.5	30.5	38.0	43 0
2.5	40.4			2.5	33.9	34.0	34.4	43.5	50.3
3.0	45.0			3.0	36.3	38.0	37.1	48.0	54.5
3.5	49 4			3.5	42.2	41.7	42.0	51.0	63.0
4.0	52.5			4.0	41.3	45.0	43.2	55 3	
4.5	59.6			4-5	45.5	48 0	46.7	61.0	
5.0		61.0		5.0	48.4	53 5	49.5		
5.5		65.7		5.5	53.0	55.0	54.0		
6 0	69.8	69.5	69 65	6.0	56 1	58 8	57.4		
6.5	73.4	74.7	74.05	6 5	59.8	62.0	60.9		
7.0	77.5	79.2	78.35	7.0	63 3	64.7	61.0		
7.5	83.8	83.0	83.4	7.5	67.5	69.0	68 3		
8.0	86.8	87.3	87 05	8.0	70 9	73.4	72 1		
8.5	90.5	90 2	90 35	8.5	73 8	76.0	75 9		
9.0	95.0	93 7	94.35	9 0	79 8	79.2	79 5		
9.5	97.7	96.3	97.0	9.5	84 8	82.5	83 6		
10.0	101.5	99 0	100.25	10.0	88 8	86.4	87 6		
10 5	107.0	103.0	105.0	10.5	93.5	89 5	91.5		
11.0	111.5	107.5	109 5	11.0	97 7	93.0	95.4		
11.5	114.0	110.5	112.3	11.5	102.0				
12.0	121.0	116.0	118.5	12.0	107 7				
12.5	125.5	120.0	122 75	12.5	111.0				
13.0	133 0	123.0	128.0	13.0	117 5				
13.5	135.0	127.0	131.0	13.5	122.5				
14.0	140.0	129.0	134.5	14.0	128.0				
14.5	144 0	136.0	140.0	14.5	134.4				
15.0	150.0			15.0	138.3				
15.5	155.0								
16.0	159 5 §								

* 85° F. Weather sultry.

† 75°-80° F. Weather clear and bright.

‡ 70° F. Weather cool and cloudy.

§ Internal discharges in intermediary transformers F′ F″.

In most of the tests the temperature and the weather are given from the record sheets. The meteorological record of the days during which tests were made is given in Table X. To check the influence of the weather, a few readings with 1″ spheres and

ironclad generator were repeated on a sultry and rainy day, but found to agree with the readings taken on a clear and bright day.

With the E. M. F. of the smooth core alternator brush discharges at the electrodes were observed by the hissing noise produced by them when the potential approached the striking

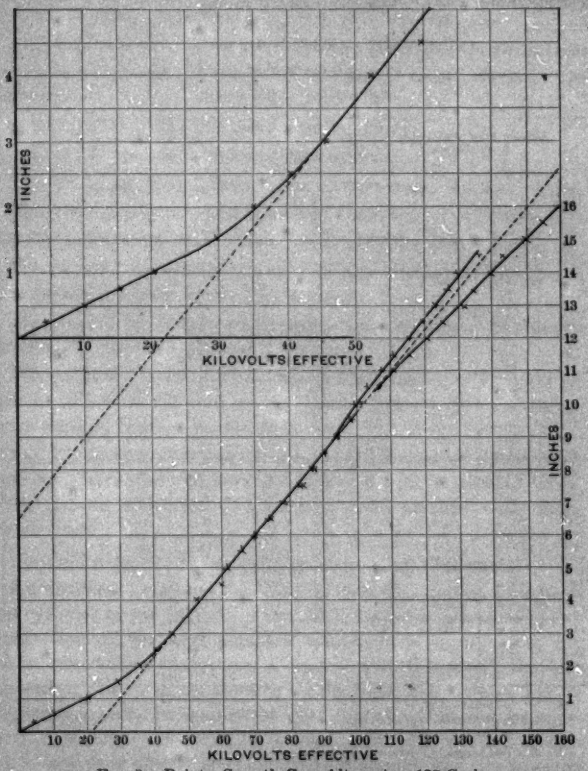

FIG. 6. Points, Smooth Core Alternator, 125 Cycles.

voltage, and at greater distance even far below this, with sharp points with smaller spheres and with parallel cylinders of smaller diameter, less pronounced with larger spheres and parallel cylinders of large diameter.

When approaching 160,000 volts effective with a smooth core alternator, electrostatic discharges took place inside of the intermediary transformers F′ and F″, apparently between primary and secondary. It seems the transformers acted as a system of six condensers in series, the two outside high-potential transformers H_1 and H_4, and the two sets of intermediary transformers F′ and F″ connected thereto with the insulation between primaries and secondaries as dielectric. At or near 160,000 volts the electro-

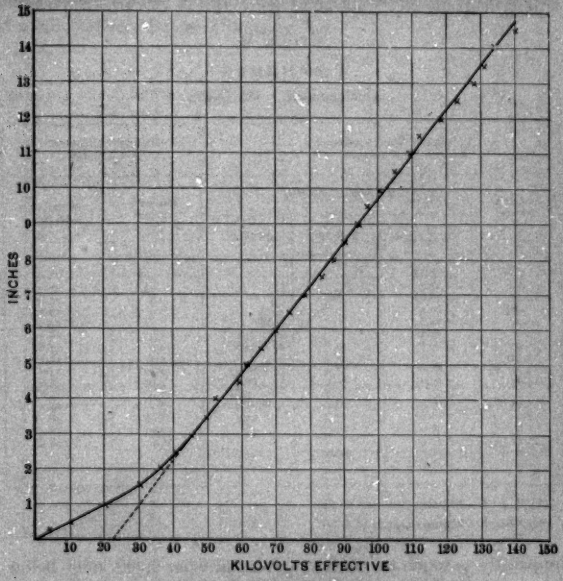

FIG. 7. Points, Smooth Core Alternator, 125 Cycles.

static stress exceeded the disruptive strength of the insulation between primary and secondary in F′ and F″. Since, however, the insulation between H_1 and H_2 resisted, no arc followed the discharge. Nevertheless, even at lower voltage, a number of intermediary transformers, F′ and F″, were pierced.

With the ironclad alternator with sharp points as well as with brass spheres, the breaking-down point was quite unstable.

Although the brass spheres were polished, amalgamated and rubbed smooth before each test with the same care as in the tests with the smooth core alternator, nevertheless the readings are far more erratic and in several instances the same disruptive voltage was found with a change of the striking distance of as much as 1″. In the record sheets all questionable readings are omitted.

With the ironclad alternator, at voltages of 80,000 and upward, almost always several heavy electrostatic discharges took place between the electrodes before the disruptive voltage was reached,

TABLE II.

1″ SPHERES.

Amalgamated. 125 Cycles.

Smooth Core Alternator: A-10-30-1500.				Ironclad Alternator: A-10-60-1500.	
Distance, inches. d:	Kilovolts; effective.			Distance, inches. d:	Kilovolts; effective.
	7-13-'96. 7-14-'96.*	7-25-'96.	Average.		7-2-'96.†
.125	8.18			.125	6.4
.25	12.9			.25	11.3
.50	18.65			.5	13.4
.585	22.4			1.0	18.2
1.47	34.8			1.523	31.3
2.0	34.7			2.0	28.2
2.97	43.6	44.8	44.2	3.0	37.8
3.51		50.0		4.0	47.7
4.01	56.5	56.4	56.45	4.97	53.8
5.03	67.0	66.0	66.5	5.97	58.0
5.5		71.0		7.0	65.8
5.99	77.0	76.4	76.7	8.0	75.1
7.0	86.8	83.6	85.2	8.97	86.2
8.0	96.0	90.6	93.3	10.0	98.4
9.03	103.5			11.0	108.5
10.02	108.7	100.6	104.62		
11.01	111.5				

* 80°-90° F. Weather clear and bright.

† 73°-80° F. Weather rainy and sultry

frequently as much as 1,000 to 1,500 and even 2,000 volts below the actual striking voltage. These discharges were never observed with the smooth core alternator, but were most marked with ½″ and 1″ spheres and with sharp points. To investigate the frequency, a Thomson transformer, that is a transformer consisting of a small number of primary and of secondary turns wound in air (without iron core) was inserted in series in the discharge circuit with sharp points as electrodes, and the secondary terminals of this Thomson transformer separated by a gap of ⅛″. When approaching the striking voltage, but before the final disruptive

discharge took place, a continuous discharge passed across the $\frac{3}{8}''$ air-gap of the secondary circuit of the Thomson transformer, showing thus the existence of an oscillatory or very high frequency effect.

Photographic investigations of the phenomena taking place during the disruptive discharge have shown that it always consists of an oscillating electrostatic discharge with at least two or three well marked oscillations followed afterwards by an arc, as I have discussed more fully in a previous article on "Photographic Study of a 150,000-volt Power Discharge."

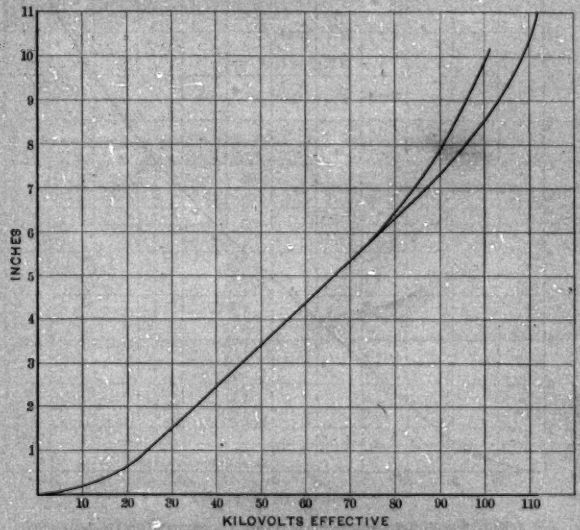

FIG. 8. $\frac{1}{4}''$ Spheres. Smooth Core Alternator, 125 Cycles.

Condenser effects were observed only once when two insulated brass disks were inserted in the path of the discharge between $\frac{1}{4}''$ spheres separated by $9\frac{1}{2}''$, and these brass disks brought very close, within $\frac{1}{2}''$ to the discharge terminals. In this case the primary current in the generator circuit rose. As above discussed, capacity within a reasonable limit has no effect on the ratio of transformation, but would show a rise of the current in the generator circuit, and no such rise was observed, except in the above mentioned case.

Regarding the effect of frequency on the striking distance, within the range of frequency easily accessible, that is from 40 to 125 cycles, no noticeable change was observed.

Of more general interest are the tests of the striking distance made with the smooth core alternator in which the wave is nearly a sine wave and the ratio of maximum to effective E. M. F. thus constant and known. The striking distance with the ironclad alternator are of interest only in so far as they show which devia-

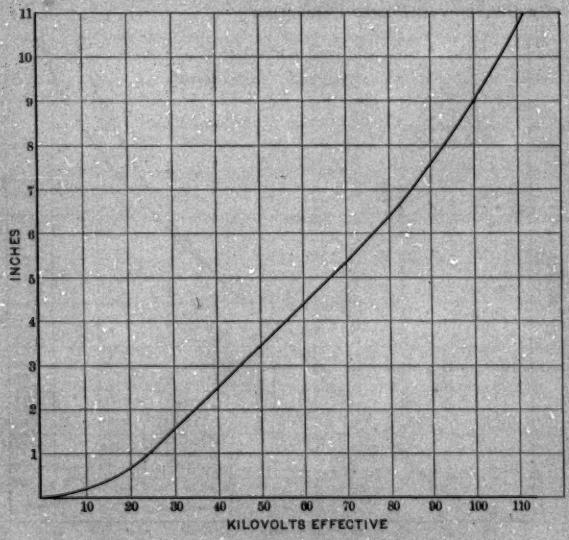

FIG. 9. ½" Spheres. Smooth Core Alternator, 125 Cycles.

tions from the sine wave curve can be expected with this type of machine most commonly in use, of sawtooth wave shape. Since, however, during the tests the voltage was varied by varying the alternator excitation and the wave shape of the ironclad changes more or less with the excitation, these curves are of comparative interest only.

Restricting thus at first our attention to the tests made with the smooth core alternator, we find the striking distance between

parallel cylinders in Tables VI and VII and Fig. 14 and 15 represented by curved lines, the striking distance increasing faster than the voltage. The curve of parallel cylinders of larger diameter resembles the lower part of the curve of cylinders of smaller diameter in increased scale, as is to be expected.

Very interesting is the curve of striking distance between sharp points, Table I and Figs. 6 and 7. It starts in a straight line, the striking distance being proportional to the voltage at the rate of 20,000 volts per inch. At about $1\frac{1}{4}''$ distance the curve bends upward and passes into a second straight line which it follows from 3″ distance upward, the distance increasing at the rate of 1″ per 8,000 volts effective. This second straight line, however, does not pass through the origin, but through a point of the voltage of 22,000 effective.

TABLE III.

¼′ SPHERES.

Amalgamated. 125 Cycles.

Smooth Core Alternator: A-10-30-1500.		Ironclad Alternator: A-10-60-1500.	
Distance, inches. d.	Kilovolts; effective. 7-23-'96. 7-14-'96.*	Distance, inches. d.	Kilovolts; effective. 7-20-'96.†
.125	8.2	.125	6.86
.25	14.6	.25	11.9
.485	22.0	.5	17.5
.94	31.3	1.0	27.9
1.5	36.1	1.5	36.5
2.0	39.4	2.0	35.4
2.5	44.4	2.97	44.0
2.90	47.9	4.0	51.2
4.0	59.4	4.95	56.1
5.0	68.0	5.94	60.0
6.06	79.0	6.97	68.8
6.97	84.0	7.97	77.5
7.97	95.0	8.97	88.0
9.0	98.3	9.97	97.3
9.93	105.5	11.03	108.8
11.0	112.0		

* 80°-90° F. Weather clean and bright.

† 75°-80° F. Weather rainy and sultry.

At very high voltages, about 100,000, the two sets of readings taken diverge from each other, the one falling below, the other passing above the straight line, as shown in Fig. 6.

The average, however, follows very closely the straight line as shown in Fig 7, where the average values of the two sets of readings are marked by crosses.

The four sets of curves representing the striking distance between spheres of various diameters have a considerable similarity with each other, the same curve repeating itself in increased scale with the increased diameter of the spheres. They differ, however, greatly from the curve of sharp points.

All the four curves start at the same slope for very small distances, of about $\frac{1}{8}''$ per 10,000 volts. Then they bend upwards and pass into a second straight line, and the middle of the bend

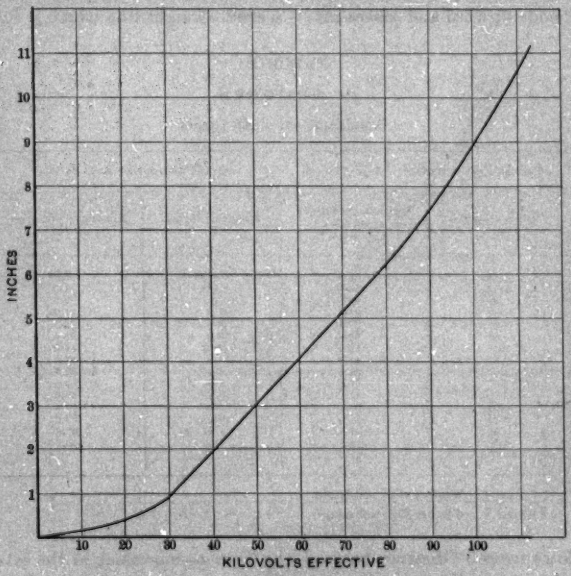

FIG. 10. $\frac{1}{4}''$ Spheres. Smooth Core Alternator, 125 Cycles.

lies at about twice the diameter of the spheres. After the bend they follow a straight line for a certain range, but bend upwards once more for high voltages and then approach rapidly the curve of sharp points. The second upward bend is marked with $\frac{1}{4}''$ and $\frac{1}{2}''$ spheres, while with $1''$ spheres the curve extends only to what is probably the end and with $2''$ spheres the beginning of the straight lined part. At very high voltages the $\frac{1}{8}''$ and $\frac{1}{4}''$

TABLE IV.

1" SPHERES.

Amalgamated. 125 Cycles.

Smooth Core Alternator: A–10–30–1500.		Ironclad Alternator: A–10–60–1500.	
Distance, inches. d.	Kilovolts; effective. 7–13–'96. 7–14–'96.*	Distance, inches. d.	Kilovolts; effective. 7–17–'96.†
.125	8.95	.125	6.75
.25	15.9	.25	11.6
.5	26.1	.485	19.6
1.0	43.6	1.00	33.3
1.47	52.8	1.188	37.2
1.94	59.2	1.97	46.0
2.97	67.7	3.0	52.5
3.99	76.0	4.0	66.7
5.0	81.2	4.99	69.2
5.03	86.5	6.0	74.7
6.61	89.0	6.99	79.5
7.28	90.0	8.2	86 0
7.99	99.0	8.99	92.0
8.99	104.5	10.0	97.5
9.99	108.7	10.97	107.0
10.94	122.0		

* 80°–90°. Weather clear and bright.
† 75°–80°. Weather clear.

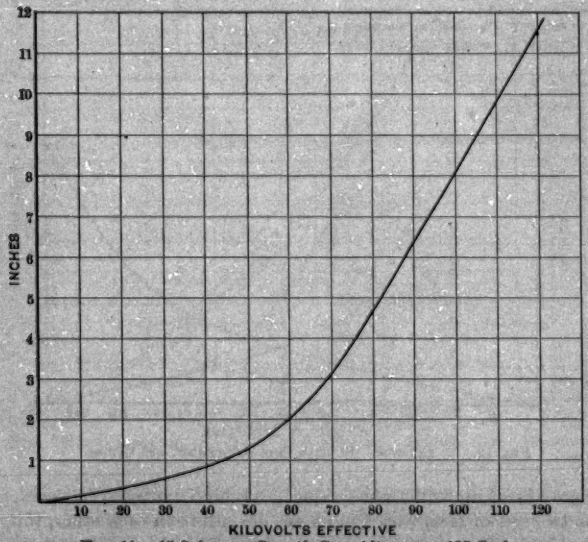

FIG. 11. 1" Spheres. Smooth Core Alternator, 125 Cycles.

spheres have practically approached each other, the 1″ spheres are coming close to the same value, while the 2″ spheres show a bend, which as seen in Fig. 8, would bring the curve beyond the

TABLE V.

2″ SPHERES.

Amalgamated. 125 Cycles.

Smooth Core Alternator: A-10-30-1500.		Ironclad Alternator: A-10-60-1500.	
Distance, inches. d:	Kilovolts; effective. 7-13-'96. 7-14-'96.*	Distance, inches. d:	Kilovolts: effective. 7-20-'96.†
.125	8.95	.125	6.67
.25	15.9	.25	12.5
.493	26.7	.50	22.6
1.08	51.0	1.03	39.3
1.454	65.2	1.50	50.8
1.688	70.8	1.95	57.8
2.25	83.8	2.51	66.7
3.0	94.0	2.99	75.0
3.438	102.0	3.5	80.3
2.952	101.3	4.0	82.7
5.08	108.0	4.49	92.0
5.60	114.5	4.97	94.9
		5.47	96.3

* 80°-90° F. Weather clear and bright.

† 75°-85° F. Weather rainy and sultry.

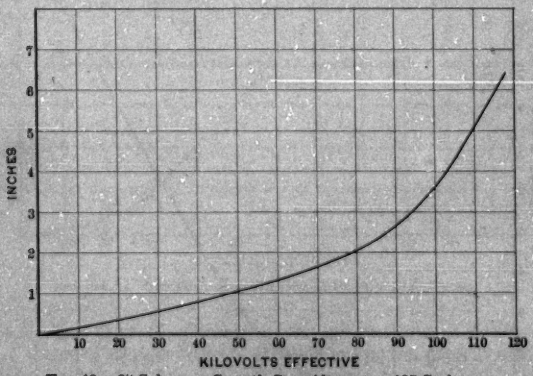

FIG. 12. 2″ Spheres. Smooth Core Alternator, 125 Cycles.

range of test up to the same value. At high voltages, however, the two sets of tests with $\frac{1}{4}$″ spheres deviate from each other, the one leaving the straight line sooner than the other. Thus their average has been plotted in Figs. 2, 9 and 13.

While the striking distances between spheres of different diameters are very different at intermediary voltages, they seem to coincide at very low and very high voltages, but differ essentially from the curve taken between sharp points, except at extremely high voltages.

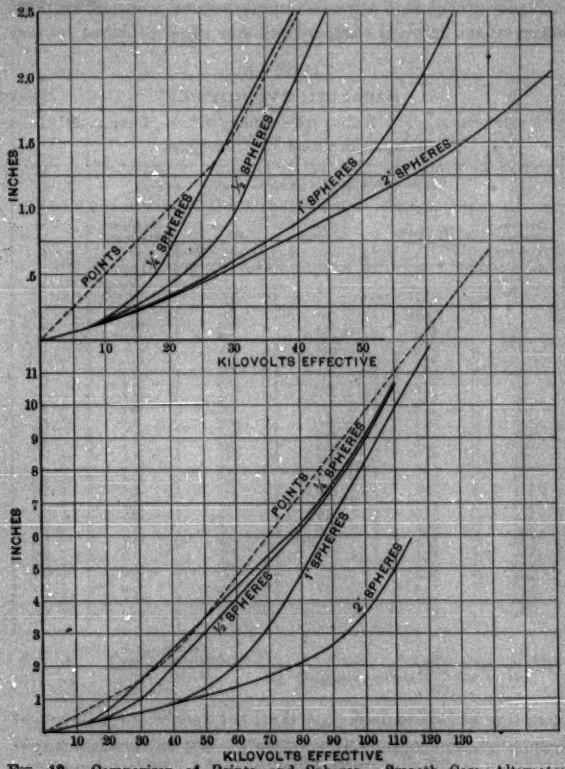

FIG. 13. Comparison of Points and Spheres. Smooth Core Alternator, 125 Cycles.

Comparing the striking distance between small spheres, and those between needle points as shown in Fig. 13, the remarkable feature is observed that the $\frac{1}{4}''$ spheres approach sharp points at

comparatively low voltages, and even intersect the curve of sharp points, so that within a certain range of voltage, from 28 to 49 kilovolts, the striking distance between $\frac{1}{4}''$ spheres is greater than between sharp points, but beyond this range these curves separate again widely, far beyond the possible error of observation, and at very high voltages approach once more. The intersection of the curve of sharp points and of $\frac{1}{4}''$ spheres is very

TABLE VI.

PARALLEL CYLINDERS.

.313″ diameter. Radius of Curvature : 87″. Length : 24″

Amalgamated brass. 125 cycles.

Smooth Core Alternator : A–10–30–1500. 4–14–06.

Distance, inches. d:	Kilovolts, effective :	Mean Gradient. g :	$\sigma\sigma_0$ *	Maximum gradient : g_0 :	Δ :
.065	4.78	73.5	.934	78.6	+ 1.5
.080	5.7	71.2	.921	77.2	+ .1
.174	10.85	62.2	.849	73.4	— 3.7
.347	18.55	53.4	.749	72.0	— 5.1
.439	23.7	53.9	.698	77.2	+ .1
.47	22.9	48.8	.685	71.2	— 5.9
.565	23.0	40.7	.643	63.3	— 13.8
.69	30.5	44 2	.600	73 7	— 3.4
.84	34 3	40.9	.556	73.5	— 3.6
.93	33.1	35.6	.533	66.8	— 10.3
1.18	41 5	35.1	.478	73.4	— 3.7
1.44	47 3	32.8	.434	75.5	— 1.6
1.72	51.4	29.9	.396	75.7	— 1.4
2.12	53.8	25.3	.357	71.0	— 6.1
2.62	65.0	24.5	.312	78.5	+ 1.4
2 82	67.8	24.1	.298	80.5	+ 3.4
3'16	71.0	22.5	.276	81.6	+ 4.5
3 18	69.8	21.0	.275	79.6	+ 2.5
3.69	72.5	19.65	.251	78.4	+ 1.3
3 72	73.3	19.7	.250	78.8	+ 1.7
4.06	75.6	18.6	.236	79.0	+ 1.9
4.09	77.4	18.95	.234	80.4	+ 3.3
4.34	77.3	17.8	.226	79.0	+ 1.9
4.53	77.7	17.15	.220	77.8	+ .7
4.84	80.0	16.5	.211	78.3	+ 1.2
5.28	83.5	15.8	.199	79.3	+ 2.2
5.46	83.7	15.35	.195	78.6	+ 1.5
5.84	93.6	16.0	.186	86.0	+ 8.9

Average : 76.0
Second average : 77.1

* $\sigma\sigma_0$ = ratio $\frac{\text{mean}}{\text{max.}}$ gradient, calculated.

interesting and quite marked, and cannot well be due to errors of observation, since either curve represents two independent sets of tests taken at different times which coincide, and the same intersection at a somewhat lower range is strongly marked also in the tests with the ironclad alternator, Fig. 17.

From all these tests it should be expected that a law on the disruptive strength of air could be derived.

The disruptive electrical discharge bears a striking analogy to mechanical rupture. Thus, as the mechanical strength of a beam remains practically unimpaired up to a certain load, and at this load suddenly falls to zero by mechanical rupture, so the resistance of an air space is practically infinite up to a certain potential, at which, under disruptive discharge, it suddenly falls down to practically nothing. In looking for a physical law represent-

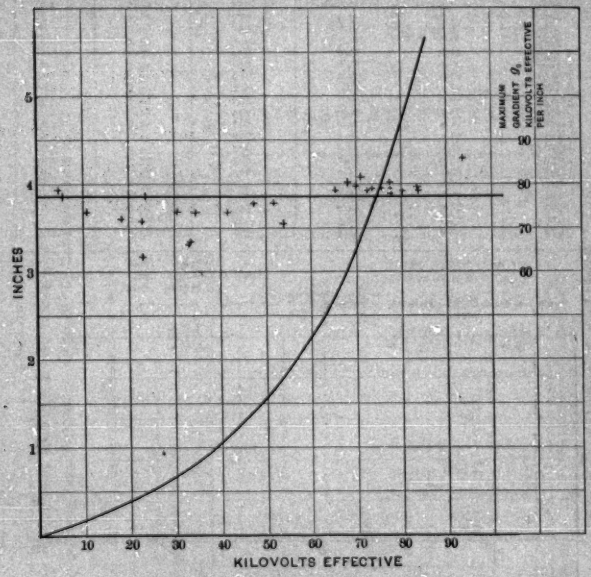

FIG. 14. Parallel Cylinder .313" Diameter. Smooth Core Alternator, 125 Cycles.

ing the electric disruptive discharge, it thus was to be investigated how far the laws of mechanical strength would represent the observed effects.

Under mechanical stress, rupture takes place as soon as the stress exceeds the breaking strain anywhere in the material, and this breaking strain is a constant of the material. Thus I investigated whether the disruptive discharge of air takes place as

TABLE VII.
PARALLEL CYLINDERS.

1.11" diameter. Radius of curvature: 140" Length: 20.
Amalgamated brass. 125 cycles.

Smooth Core Alternator: A-10-30-1500. 4-6-96.

Distance, inches. d:	Kilovolts, effective.	Mean Gradient. g:	$\sigma\sigma_0$*	Maximum gradient. g_0:	Δ:
.094	5.9	62.7	.973	64.5	+ 4.5
.109	7.08	54.4	.969	66.5	+ 6.5
.172	9 94	57.8	.951	60.8	+ .8
.188	11 6	61.7	.947	65.2	+ 5 2
.320	19.2	60.0	.914	65.6	+ 5.6
.407	21.6	53.0	.892	59.4	− .6
.492	27.0	55.0	.874	63.0	+ 3.0
.891	41.0	46.0	.797	57.8	− 2.2
1.31	52.0	39.7	.730	54.5	− 5.5
1.49	60.5	40.6	.704	57.0	− 2.4
1.78	66.2	37.2	.667	55.8	− 4.2
1.79	68.4	38.3	.666	57.6	− 2.4
2.06	74.0	36.0	.637	56.5	− 3.5
2.19	79 0	36.0	.623	57.7	− 2.3
2.37	84.6	35.7	.605	59.1	− .9
2.46	86.7	35.3	.597	59.2	− .8
2.54	93.0	36.5	.589	62.0	+ 2.0
2.90	98.5	34.0	.559	60.8	+ .8
3.19	103.7	32.5	.536	60.7	+ .7
3.72	118.4	31.8	.502	63.5	+ 3.5

Average: 60.4
Second average: 59.6
Mean: 60.0

* $\sigma\sigma_0$ = ratio $\frac{\text{mean}}{\text{max.}}$ gradient, calculated.

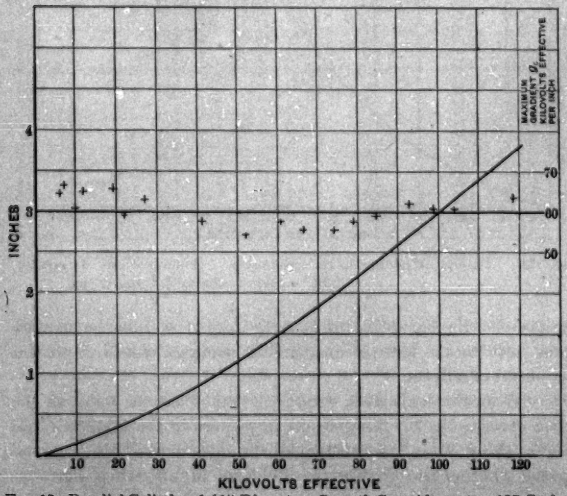

FIG. 15. Parallel Cylinder, 1.11" Diameter. Smooth Core Alternator, 125 Cycles.

TABLE VIII.

Brass Disks of $\frac{6}{32}$′ Thickness, in Path of Discharge of ¼′ Spheres, at Right Angles Thereto.

Spheres Amalgamated. Disks suspended by three silk threads. 125 cycles.

Smooth Core Alternator: A-1e-30-1500. 7-22-'96.

Number of disks.	Diameter of discs, inches.	Distance between spheres and disks. Inches.	Distance between spheres and disks. Per cent.	Total distance between spheres. *a*.	Net air distance between spheres. *b*.	Air distance between spheres around edge of disk. *c*.	Kilovolts, effective.	Δ_a	Δ_b	Δ_c*
				$8\frac{1}{32}$		.....	98			
				$8\frac{3}{32}$		.....	99			
				$8\frac{5}{32}$		.....	100	(from Table II.)		
				$9\frac{1}{32}$		.....	101			
				9.185		.....	101.3			
				10.75		.....	110			
1	15	$4\frac{1}{32} + 4\frac{1}{32}$	.50 + .50	$8\frac{1}{32}$	$8\frac{1}{32}$	.....	123.5	23.5	24.5	
1	10	$4\frac{1}{32} + 4\frac{1}{32}$	.50 + .50	$8\frac{1}{32}$	$8\frac{1}{32}$	.....	126	26	27	
1	10	$3 + 5\frac{1}{32}$	.34 + .66	$8\frac{1}{32}$	$8\frac{1}{32}$	.....	124	24	25	
1	10	$1\frac{13}{16} + 7\frac{7}{16}$	.19 + .81	$8\frac{1}{32}$	$8\frac{1}{32}$	.. ...	116	16	16	
2	10	$2\frac{1}{16} + 2\frac{6}{16} + 2\frac{1}{16}$	.33 + .33 + .33	$9\frac{1}{32}$	$8\frac{1}{32}$	.....	127	26	28	
2	10	$\frac{3}{16} + 7\frac{5}{16} + \frac{3}{16}$	.085 + .83 + .085	$9\frac{1}{32}$	$8\frac{1}{32}$	.....	94.3†	− 6.7	− 4.7	
3	10	$\frac{3}{16} + 3\frac{1}{16} + 3\frac{1}{16} + \frac{3}{16}$	.087 + .413 + .413 + .087	$9\frac{1}{32}$	$8\frac{1}{32}$	.....	94.7†	− 6.3	− 3.3	
1	6	$4\frac{1}{32} + 4\frac{1}{32}$	.50 + .50	$8\frac{1}{32}$	$8\frac{1}{32}$	10.75	115.5	15.5	16.5	5.5
1	2	$4\frac{1}{32} + 4\frac{1}{32}$	.50 + .50	$8\frac{1}{32}$	$8\frac{1}{32}$	9.185	108.5	8.5	9.5	7.2

* Δ_a = kilovolts observed minus kilovolts required for distance *a*. Δ_b = kilovolts observed minus kilovolts required for distance *b*. Δ_c = kilovolts observed minus kilovolts required for distance *c*.

† Decided capacity effect, with increase of primary current, in generator circuit, to 8 amperes.

soon as the dielectric stress, that is the electrostatic gradient anywhere in the path of the discharge exceeds a certain constant value, which would be called the "dielectric strength of air" in analogy to the "mechanical strength" of a material. The mechanical strength of a material is tested by subjecting it to a uniform mechanical stress, the usual test of tensile strength by

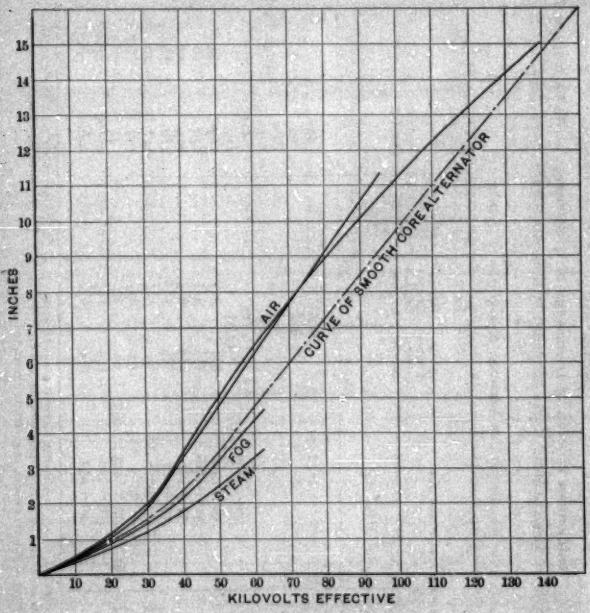

FIG. 16. Points in Air, Fog and Steam at Atmospheric Pressure. Ironclad Armature, 125 Cycles.

pulling a test piece of known cross-section to pieces. The analogous test of the dielectric strength of air would require a uniform electrostatic field, that is, parallel plates of perfect polish as electrodes. This is obviously not feasible. Thus I chose a shape of electrodes giving a relatively simple, although not uniform electrostatic field.

Between spheres, the lines of electrostatic force and equipotential surfaces are curves and surfaces of higher order. Between parallel cylinders, however, lines of force and equipotential lines are two linear systems or pencils of circles or rather cylinders, that is, all lines of electrostatic force are circles intersecting each other in two points, the fundamental points of the pencil, and all

TABLE IX.

Wet Plank Parallel to Discharge of ¼" Spheres.

Grain parallel to discharge. Mohawk river water. Spheres amalgamated.

125 cycles.

Smooth Core Alternator: A–10–30–1500. 7–22–96.

Distance, inches, between: Spheres:	Sphere and plank.	Kilovolts, effective:	Struck to:	Average Kilovolts,* effective. 9	Δ_a:	Δ_b:	Δ_c:†
6	∞	76.5	—	—			
8	"	93	—	—			
10	"	105.6	—	—			
12	"	118	—	—	From	Table 2.	
4 1/16	"	58	—	—			
6 1/16	"	77.7	—	—			
9	"	99.5	—	—			
10 3/16	"	120.5	—	—			
6	2	67.7	plank.	69.5	7.0	11.9	18.5
"	2 1/16	71.3	spheres.				
"	2 1/8	71.3	"				
"	3	72.3	"				
8	2 1/16	78	plank.	82.1	10.9	4.4	15.3
"	3	80.7	"				
"	3 1/16	83.5	spheres.				
"	3 1/8	86.5	"				
10	3 1/16	88	plank.	103	2.6	3.5	6.1
"	4 1/16	102	"				
"	4 1/8	104	spheres.				
"	4 3/16	105	"				
12	5 1/16	116.5	plank.	117.3	.7	6.8	7.5
"	5 1/8	119.5	"				
"	5 3/16	115	spheres.				
"	5 1/8	121	"				

* That is, voltage where the direct path between the spheres, and the path over the wet plank have the same dielectric strength.

† Δ_a = kilovolts of direct discharge between spheres, minus 9.

Δ_b = 9 minus disruptive voltage between spheres at twice the distance of wet plank.

$\Delta_c = \Delta_a + \Delta_b$

equipotential lines are circles intersecting the first set at right angles, and containing the cross section of the cylinders as one pair.

This being the relatively simplest case, it was chosen for the investigation. Obviously the length of the parallel cylinders

could not be infinite, and therefore, to avoid the influence of the ends of the cylinders, they were given a slight backward curvature of a very large radius compared with the diameter of the cylinders and their distance, as shown in Fig. 18, which represents the relative proportions of the larger cylinders at the highest observed striking distance. The effect of this curvature was investigated and correction made therefor.

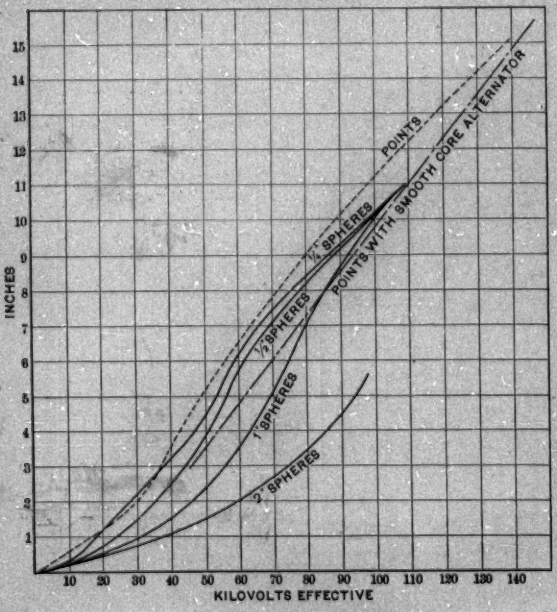

FIG. 17. Comparison of Points and Spheres. Ironclad Armature, 125 Cycles.

From the distance between the cylinders d and the striking voltage V, the mean electrostatic gradient $g = \frac{V}{d}$ was calculated and is given in the third column of Tables VI. and VII. in kilovolts per inch. From diameter and distance of cylinders, the ratio of mean to maximum gradient σ was calculated, and by

TABLE X.

METEOROLOGICAL RECORD AT DATE OF TESTS.

Date. 1896.	TEMPERATURE ° C:					BAROMETER, ":		SKY:			Tests made.
	Preceding nightly minimum.	8 h morning.	Daily maximum.	9 h evening.	Following nightly minimum.	8 h morning.	9 h evening.	Forenoon.	Noon.	Afternoon.	
April 6	−2	+1.5	+11.5	+8	+.5	29.9 "	29.8"	cloudless		almost cloudless	1.11" cylinders, smooth core.
" 14	——	17	25	17	——	29.55"	29.7"	cloudless	cloudy	almost cloudless	.313" " "
July 13	16	——	30	24.5	23	29.7 "	29.6"	almost cloudless	cloudy	almost cloudless	¼", ½", 1", 2" spheres.
" 14	23	25	29	23	18.5	29.65"	29.65"	cloudy		almost cloudless	Smooth core.
" 17	13	17.5	26	20	15.5	29.8 "	30.0"	almost cloudless			Needle points, smooth core.
" 20	20	21.5	25.5	23.5	21	29.9 "	29.7"	rain	rain	cloudy	" spheres, ironclad. ¼", ½", 2" spheres, ironclad.
" 21	21	25	31	25	22.5	29.77"	29.8"	almost cloudless			Needle points, smooth core.
" 22	22.5	23.5	31.5	23	19	29.8 "	29.5"	almost cloudless		rain and thunderstorm	Brass disks and wet plank.
" 24	15.5	18	20.5	16	15.5	29.75"	29.5"	cloudy	rain	rain	Needle points, ironclad.
" 25	15.5	18	26.5	18.5	15	29.62"	29.8"	almost cloudless			¼" spheres, smooth core.

dividing the observed mean gradient g by the factor σ, the maximum gradient $g_o = \frac{g}{\sigma}$ would be derived for parallel cylinders.

To correct for the backward curvature of the cylinders, of radius r_o, the ratio of mean to maximum radient was calculated for the distance d but a diameter of cylinders of $2r_o$. Since σ_o differs very little from one, being at the highest distance observed with large cylinders $\sigma_o = .994$ and with small cylinders $\sigma_o = 0.893$.

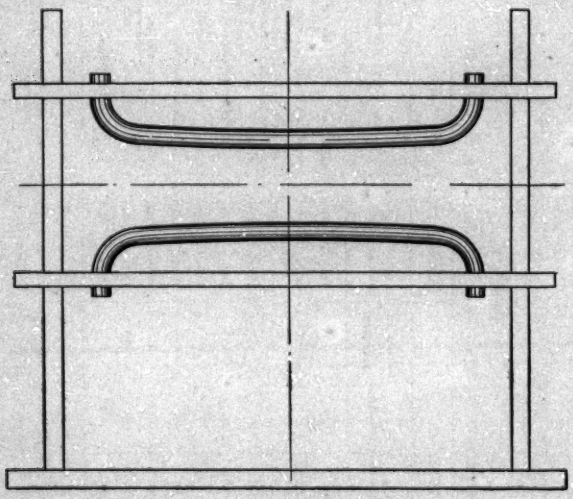

FIG. 18.

by dividing with σ_o the correction was far within the errors of observation.

Thus the maximum gradient g_o was derived from the observed mean gradient $g = \frac{V}{d}$ as: $g_o = \frac{g}{\sigma\sigma_o}$.

The calculation of σ and σ_o is as follows:

Let in Fig. 19 be represented the cross-section of the two parallel cylinders of equal diameter.

Let x = abscissæ with center 0 between the parallel cylinders as origin.

y = ordinates of one line or circle of electrostatic force of radius R.

d = distance inside between parallel cylinders.

r = radius of parallel cylinders.

p = distance of fundamental points A and A^1 of cylindrical pencil (common point of intersection of all lines of force) from origin.

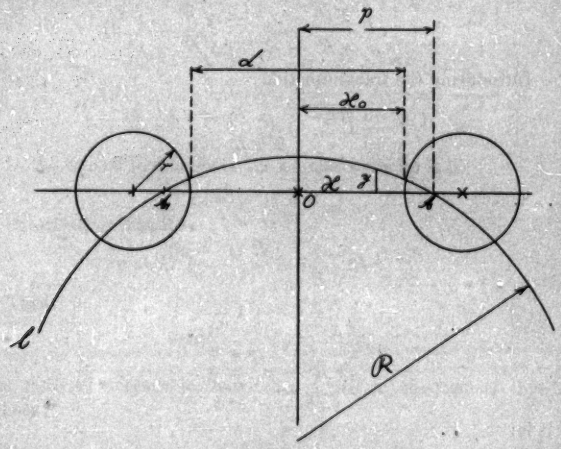

FIG. 19.

At the surface of the cylinders, it is:

$$y_0 = \sqrt{R^2 - x_0^2} - \sqrt{R^2 - p^2};$$

in general it is:

$$y = \sqrt{R^2 - x^2} - \sqrt{R_2 - p^2};$$

it is:

$$p^2 = x_0 (x_0 + 2r).$$

Let g_0 = electrostatic stress at the cylinder surface in the central line, that is at x_0; then it is:

$$g_x = \frac{y_0}{y} = \text{electrostatic stress at point } x \text{ of central line,}$$

if the line of force 1 is infinitely near the central line, that is, if:

$$r = \infty .$$

Thus the ratio of average electrostatic stress g to maximum stress g_o is:

$$\sigma = \frac{g}{g_o} = \frac{1}{x_o}\int_o^{x_o} \frac{y_o}{y}\, d_x .$$

Substituting, it is:

$$\sigma = \frac{1}{x_o}\int_o^{x_o} \frac{\sqrt{R^2 - x_o^2} - \sqrt{R^2 - p^2}}{\sqrt{R^2 - x^2} - \sqrt{R^2 - p^2}}\, d_x .$$

Introducing the transformation:

$$\sqrt{R^2 - x^2} = R - u\,x,$$

$$x = \frac{2\,u\,R}{1 + u^2},$$

$$d = \frac{2\,R\,(1 - u^2)}{(1 + u^2)^2},$$

$$\sqrt{R^2 - x^2} = \frac{R\,(1 - u^2)}{(1 + u^2)},$$

$$u = \frac{R}{x}\left\{1 - \sqrt{1 - \frac{x^2}{R^2}}\right\},$$

$$\sqrt{R^2 - x_o^2} = a,$$

$$\sqrt{R^2 - p^2} = b,$$

it is:

$$\sigma = \frac{2\,R\,(a - b)}{x_o}\int_o^{x_o} \frac{(1 - u^2)\, d_x}{(1 + u^2)\,\{(R - b) - (R + b)\,u^2\}}.$$

Substituting:

$$R - b = c^2,$$

$$R + b = g^2,$$

$$\delta = \frac{2\,R\,(a - b)}{x_o\,(g^2 + c^2)}\int_o^x \left\{\frac{1 + u^2}{2} + \frac{g^2 - c^2}{2\,c}\cdot\frac{}{g\,u + c} - \frac{g^2 - c^2}{2\,c}\cdot\frac{}{g\,u + c}\right\} d\,x.$$

$$\frac{2\,R\,(a - b)}{x_o\,(g^2 + c^2)}\ \text{arc tan}\ u + \frac{g^2 - c^2}{2\,g\,o}\ l\,g\,\frac{g\,u + c}{g\,u - c}\Bigg|_o^{x_o}$$

or substituting:

$$\sigma = \frac{R}{x_0}\left\{\sqrt{1-\frac{x_0^2}{R^2}}-\sqrt{1-\frac{p^2}{R^2}}\right\}\Big/ 2 \arctan\frac{R}{x}\left\{1-\sqrt{\frac{x^2}{R^2}}+\right.$$

$$\left.\frac{R}{p}\sqrt{1-\frac{p^2}{R^2}}\, lg\, \frac{p\left\{1-\sqrt{1-\frac{x^2}{R^2}}\right\}+x\left\{1-\sqrt{1-\frac{x^2}{R^2}}\right\}}{p\left\{1-\sqrt{1-\frac{x^2}{R^2}}\right\}-x\left\{1-\sqrt{1-\frac{p^2}{R^2}}\right\}}\right|_{x_0} =$$

$$\left.\begin{array}{l}\frac{R}{x}\left\{\sqrt{1-\frac{x^2}{R^2}}-\sqrt{1-\frac{p^2}{R^2}}\right\} 2 \arctan\frac{x^2}{R}\left\{1-\sqrt{1-\frac{x^2}{R^2}}\right\}+ \\ \frac{R}{p}\sqrt{1-\frac{p^2}{R^2}}\, lg\, \frac{x_0\left\{1-\sqrt{1-\frac{p^2}{R^2}}\right\}+p\left\{1-\sqrt{1-\frac{x^2}{R^2}}\right\}}{x_0\left\{1-\sqrt{1-\frac{p^2}{R^2}}\right\}-p\left\{1-\sqrt{1-\frac{x^2}{R_2}}\right\}}\end{array}\right\}^{p}.$$

In the central line between the parallel cylinder it is:

$$R = \infty.$$

Substituting this, it is:

$$\sigma = \sqrt{1-\frac{x^2}{R^2}} = 1-\frac{1}{2}\frac{x^2}{R^2}-\frac{1}{8}\frac{x_0^4}{R^4}.$$

Thus:

$$\sigma = \frac{p^2-x^2}{2\,p\,x_0}\, lg\, \frac{p+x_0}{p-x_0},$$

the ratio of average to maximum strain in the central line, where:

$$x_0 = \frac{d}{2},$$

$$p^2 = x_0\,(x_0 + 2\,r).$$

Substituting this, it is:

$$\sigma = \frac{r}{p}\, lg\, \frac{p+x}{p-x},$$

and if the distance x_0 is very small compared with the radius r, it is:

$$\sigma = \frac{r}{p}\, lg\, \frac{p+x_0}{p-x_0}$$

$$= \frac{2r}{p}\left(\frac{x_0}{p}+\frac{x_0^3}{3p^3}+\frac{x_0^5}{5p^5}\right),$$

or, since:

$$p^2 = x_0 (x_0 + 2r,$$

approximately:

$$\sigma = 1 - \frac{x_0}{2r}.$$

If now

r = diameter of cylinders,

r_0 = ratio of curvature of cylinders (r_0 being very large compared with r and x_0), it is:

$$\sigma = \frac{r}{p} lg \frac{p + x_0}{p - x_0},$$

$$\sigma_0 = 1 - \frac{x_0}{2 r_0},$$

and

$$\sigma \sigma_0 = \frac{r}{p}\left(1 - \frac{x_0}{2 r_0} lg \frac{p + x_0}{p - x_0}\right),$$

the ratio of the maximum to the mean electrostatic gradient within the path of the discharge.

Thus, if g = mean gradient,

$$g_0 = \frac{g}{\sigma \sigma_0}$$ is the maximum gradient.

With the cylinders used it was:

$$r = .1565 \text{ and } .555 \text{ respectively.}$$

Herefrom the numerical values of correction factor $\sigma\sigma_0$ and maximum gradient g_0 are calculated and given in Tables VI and VII.

With the large cylinders, which were investigated first, the values of the maximum dielectric gradient g_0 were found over the whole range from the lowest to the highest voltage to average 60. The difference of individual values of g_0 from the average is not greater, but rather less than found under similar conditions in tests of tensile strength in mechanics. They are, however, greater than can be accounted for by the errors of observation, thus showing the existence of some secondary effect disturbing the results. The values of g_0 show no noticeable tendency of deviation from constancy for very high or very low voltages, and appear rather low in the middle, high at either end of the curve.

The tests with the smaller cylinders gave an equally good agreement between the values of the maximum gradient g_0.

However, comparing both sets of tests we found that the "dielectric strength of air" averages 60 with the 1.1″ cylinders but averages 77 with the .313″ cylinders. Thus, while the max-

imum gradient is fairly constant with the same size of cylinders and different distances, it differs considerably with different sizes of cylinders, or in other words the dielectric strength of air as found in this way is not a constant, as the tensile strength of mechanics, but varies with the conditions.

Some further results on the question of the constant dielectric strength of air can be derived from the tests with points and with spheres as terminals.

Between mathematical points the dielectric field of force is similar regardless of the distance. Thus with the needle points used, for all distances great compared with the radius of curvature of the needle points, that is already the smallest distance investigated, the striking distance should be proportional to the voltage if it depends upon the maximum gradient. This, however, is not the case.

Between mathematical points even with the lowest voltage the gradient at the points is infinite. Thus with needle points very small voltages should strike over great distances. This is not the case either, but the curve of needle points does not differ much from that of $\frac{1}{4}''$ spheres. This leads us to one of the phenomena obscuring the true electrostatic gradient at the moment of disruption, the brush discharge.

Long before the disruptive voltage is reached between needle points and smaller spheres or cylinders, a brush discharge issues from the terminals into the surrounding air. The space surrounding the electrodes is filled with an infinite number of violet streamers issuing from the electrodes under a hissing noise and being constantly in motion, and this space is thus more or less broken down dielectrically, so that, when the final disruptive discharge takes place, the terminals of the discharge are no longer the original metal terminals, but the whole space covered by the brush discharge is more or less to be considered as discharge terminals.

An attempt was made to investigate this phenomenon, thus: Instead of the amalgamated metal cylinders of diameter D and distance d, two cylinders were substituted of the diameter $D + x$ and distance $d - 2x$ and for the latter cylinders the mean gradient $g = \frac{V}{d-2x}$, the ratio of mean to maximum gradient $\sigma = \sigma_0$ and this the maximum gradient g_0 calculated for various values of x.

Representing then the length of the brush discharge by x, with constant D, d, and V, g_o decreases with increasing x, reaches a minimum and then increases again; or inversely, at a constant maximum gradient g_o, with increasing length of brush discharge, the voltage required to give this gradient g_o will increase, reach a maximum and then decrease again. Thus if the terminal voltage is gradually increased, as soon as the maximum gradient at the terminals exceeds the breaking strain, a brush discharge will issue and thereby enlarge the size of the terminals by the space covered by the brush discharge, but at the same time decrease the maximum gradient at the surface of the effective terminals, that is, the space covered by the brush discharge, until the critical point is reached beyond which a further expansion of the brush discharge increases the gradient at its surface again. At this point a disruptive discharge will take place.

No satisfactory results were found, however, by this method, probably for the reason that:

1st. The brush discharge at high voltage does not cover a well defined distance, but consists of an infinite number of streamers issuing from the electrodes to a greater or lesser distance and being in rapid and violent agitation.

2nd. The space covered by the brush discharge is only partially broken down, that is while not having the dielectric strength of air before the break-down, its dielectric strength has not fallen to zero yet, as after the discharge. It thus represents electrostatically a similar condition as mechanically a medium under strain exceeding the elastic limit of the material. Under mechanical stress, rupture does not always take place as soon as the strain anywhere in the cross-section exceeds the breaking strain.

Comparing, for instance, a beam of the cross-section A in Fig. 20 with that in B of the same figure, which latter differs only by the addition of the two small ribs a on top and bottom, it is obvious that B has no lesser strength than A. But at the extremities of the ribs a the breaking strain is exceeded at a load far below that which would break down beam A. The result is that in the two thin ribs a, cracks are formed or the material strained beyond the elastic limit, and only at much greater load the break-down of the beam occurs.

The tests made with spheres allows some further conclusions. The electrostatic field of force between two spheres of diameter D and distance d is similar to the field between spheres of diame-

ter n D and distance n d. Thus the ratio between mean and maximum gradient is the same. Hence, if disruptive discharge takes place at the same maximum gradient, in the latter case n times the striking voltage would be required.

Since tests with four different diameters of spheres were made, this feature is easy to investigate. In the following a number of sets of readings are given abstracted from Tables II to V: Values are chosen in that range of voltage where the four curves differ most. The first column gives the diameter of the sphere

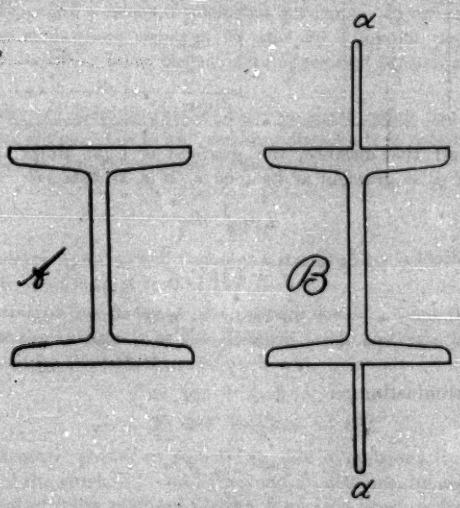

FIG. 20.

D, the second column the striking distance d' chosen proportional to the sphere diameter, the third column the striking voltage, and the fourth column the ratio a of mean gradients of subsequent lines. As seen, while constant dielectric strength of air would require a ratio $a = 1$, the ratio differs herefrom. This difference could not well be accounted for by a brush discharge depending upon the electrostatic gradient, and thus some further secondary effects are probable.

Diameter of spheres, inches. D.	Distance of spheres, inches. d.	Striking voltage, kilovolts. V.	Ratio of gradients. a.
1	8	98.7	
½	4	59	1.194
¼	2	34	1.152
1	6	87	
½	3	49.3	1.134
¼	1½	29	1.180
1	4	75.4	
½	2	40	1.060
¼	1	23.6	1.165
2	6	115	
1	3	68.5	1.194
½	1½	35.4	1.035
¼	¾	21	1.186
2	4	103	
1	2	59.5	1.135
½	1	30	1.010
¼	1½	17.3	1.154
		Average.....	1.155

At half the distance, the gradient is 1.135 times as great. Thus, if g_1 = gradient at distance d_1, it is, at distance:

$$d = \frac{1}{2_n}\, d_1,$$

the gradient:

$$g = 1.135^n\, g_1,$$

thus:

$$l\underline{d} = l\underline{d_1} - n l\underline{2},$$

$$l\underline{g} = l\underline{g_1} + n l\underline{1.135},$$

hence, eliminating n:

$$.183\, l\underline{d} + l\underline{g} = .183\, l\underline{d_1} + l\underline{g_1},$$

$$g\, d^{.183} = g_1\, d_1^{.183},$$

or:

$$g = \frac{a}{d^{.183}},$$

$$V = a\, d^{.817},$$

where:

$$a = g_1\, d_1^{.183} = \frac{V_1}{d_1^{.817}},$$

or:

$$d = b\, V^{1.22},$$

where:

$$b = \frac{d_1}{V_1^{1.22}}.$$

This gives a parabolic law of striking distances in uniform field.

According hereto the curve of striking distances should approach the zero point horizontally, that is for very small voltages the electrostatic gradient should approach infinity and the striking distance zero. This, however, is not the case, since even for very small voltages the striking distance and the gradient are finite. On the contrary, in the tests with spheres the gradient for very small distances approaches 75 to 80, about the same value as the dielectric strength of air as found with small cylinders. Hence, if a constant dielectric strength of air exists it would be somewhere between 60 and 80 kilovolts per inch. Some results are in favor but some in opposition to such a hypothesis.

The curve of striking distance between sharp points has a very interesting shape. Up to $1\frac{1}{2}''$ striking distance it is proportional to the voltage, and represented by the equation,

$$d = .05\ V$$

or

$$V = 20\ d,$$

then it curves upwards and becomes a straight line again from $3''$ upward to the limits of test, at $14\frac{1}{2}''$.

This straight line does not pass through the origin but through a point at the voltage 22 or the distance $-2\frac{3}{4}$, and can thus be represented by the equation,

$$d = .125\ V - 2.75,$$

or,

$$V = 8\ d + 22.$$

This feature points to something like a spurious "counter E. M. F. of the spark." That is, beyond $3''$ distance, by assuming a potential of 22 as required for the discharge passing from the terminals to the air, the remaining voltage is proportional to the distance. For low voltages the curve leaves the straight line and slopes towards the origin, becoming straight again for still lower voltages.

This shape of curve bears a striking similarity to the volt-ampere curve of the electric arc or to that of electro-chemical polarization.

In the electric arc under constant conditions the voltage is approximately proportional to the current plus a constant value, the spurious counter E. M. F. of the arc.

In an electrolyte the current is proportional to the voltage for small voltages below the counter E. M. F. of polarization. Approaching the latter the current rises beyond proportionality, that is, the volt-ampere curve bends upwards, and ultimately turns into a second straight line, when the dissociation voltage V_0 is passed, which latter straight line does not pass through the origin, however, but through the point $C = 0$, $V = V_0$, and thus can be represented by

$$C = a\ V + V_0.$$

Thus the theoretical volt-ampere curve of the electrolyte is of the same shape as the distance-volt curve between points.

Since with alternating E. M. F. the current in the electrolyte below the dissociation potential is essentially in quadrature with the E. M. F. the current over the whole range can be represented by the theoretical equation,

$$C = \sqrt{a^2\ V^2 + b^2\,(V - V_0)^2}$$

where the latter term appears only beyond $V = V_0$. It is of interest to note that the curve of striking distance between sharp points over the whole range from the origin to the limits of observation can be represented by

$$d = \sqrt{.05^2\ V^2 + .115^2\,(V - V_0)^2}.$$

This equation represents the curve absolutely as shown in the following comparison, where the first column gives the voltages, the second the striking distance taken from test, and the third the striking distances as calculated from this formula:

$$d = \sqrt{d_1^2 + d_2^2}$$

$$d_1 = .05\ V$$

$$d_2 = .115\ (V - 27.5.)$$

V:	d calc:	d obs:
10	.5	.5
20	1.0	1.0
27.5	1.38	1.38
30	1.53	1.53
60	4.8	4.8
90	8.5	8.5
120	12.35	12.25
150	16.0	16.0

The magnitude of this spurious counter E. M. F. or transition resistance of the spark is the same as that at which the brush discharge is formed. Not the electrostatic glow of bluish color which under favorable conditions can be observed already below 1,000 volts, but the real brush discharge consisting of violet streamers issuing from the terminals into the surrounding air.

The tests with spheres show also more or less marked the appearance of straight lined parts in the curve, passing through a point differing from the origin.

The most interesting feature in connection therewith is, however, if an electric conductor as a brass disk is inserted into the path of the discharge.

A considerable number of tests were made and are recorded in table 8, by inserting brass disks of different diameters and in different positions and different numbers into the path of the discharge between $\frac{1}{4}''$ spheres. The first column of table 8 gives the number of disks used, the second the diameter, the third and fourth the position of the disks between the spheres, the fifth and sixth the total distance and the net distance between the spheres and the seventh the striking voltage. The last three columns give:

As Δ_a the difference in the observed voltage and the striking voltage between $\frac{1}{4}''$ spheres at the same total distance without disks between them.

As Δ_b the difference in the observed voltage and the striking voltage between $\frac{1}{4}''$ spheres at a distance equal to the net air distance between spheres and disks.

As Δ_c the difference in the observed voltage and the striking voltage between $\frac{1}{4}''$ spheres at the same distance as in the test, but in a line over the edge of the disk.

It is of interest to note that the insertion of an electric conductor, as a brass disk, increases the voltage required to strike between the spheres, although the net air distance is reduced by the thickness of the disk.

Thus with one $10''$ disk midway between $\frac{1}{4}''$ spheres at about $9''$ distance, 126 kilovolts are required to strike across, while after withdrawing the disk 100 kilovolts will strike; or in other words, at voltages between 100 on 126 kilovolts between $\frac{1}{4}''$ spheres and $9''$ distance in air, a disruptive discharge will take place, but will not take place any more if an electric conductor as a brass disk is inserted between the spheres.

The disks were suspended and insulated by three silk threads. Between 10″ and 15″ disks, no difference seems to exist, but 6″ disks increase the striking voltage less, and still less 2″ disks. In either of the two last cases, however, the striking voltage is higher than would be required for a stroke from sphere to sphere around the edge of the disk.

This phenomenon could be explained by the fact that two sparks of half the length require a higher voltage than one spark of full length. This explanation, however, does not explain the observations on the 6″ and 2″ disks, where the voltage was higher than necessary for a single stroke through air around the edge of the disk, by 5.5 and 7.2 kilovolts respectively. If the disk is not midway between the spheres, but nearer to one side, the increase of voltage is less.

Two disks equidistant from each other and the spheres, increase the voltage by the same amount as one disk in the center, and less if they are nearer to the spheres, and disks very close to the spheres even reduce the striking voltage, under marked capacity effects. The same applies to three disks.

That means, if the disks are very close to the electrodes they are probably brought in perfect electric connection therewith by the brush, and thus form a new set of electrodes.

The increase of voltages due to the insertion of an electric conductor is from 23.5 to 27 kilovolts, that is about the same as the spurious counter E. M. F. of the spark, and would thus well agree with the assumption of such a phenomenon, since the insertion of a conductor increases the number of transitions from metal to air by two, and thus should require an increase of the striking voltage by the counter E. M. F. of the spark.

The next table, No. 9, gives tests of striking distances between ¼″ spheres when an electric conductor, as a plank wetted with river water, is approached parallel to the discharge line. The effect thereof is a change of the striking voltage below that required for twice the distance between the spheres, but beyond that required for twice the distance between spheres and plank. That is, when the plank is approached so far that the single discharge between the spheres and the double discharge from spheres to plank have the same dielectric strength, the striking voltage is intermediate between that corresponding to the length of the direct discharge and a distance equal to the sum of the two discharges from spheres to plank. The difference in voltage

which would be required for direct discharge between spheres and for a discharge equal to twice the distance of the plank is 18.5 and 15.3 kilovolts respectively in the first two cases, less in the last two, where probably the resistance of the plank as an imperfect conductor was noticeable. Making allowance for the latter, we find again a voltage of the magnitude of the spurious counter E. M. F. of the spark, as discussed under sharp point.

Tests with the ironclad alternator between sharp points in air, in fog and in steam, point also to such a counter E. M. F. of the spark, or lesser magnitude with the ironclad alternator, due to the greater ratio of maximum to effective E. M. F.

These sets of tests with brass disks and wet plank were suggested by some observations made three years ago with an ironclad alternator and recorded on table I.

To investigate whether the dielectric strength of air was reduced in fog, as occasionally suggested at that time, a steam pipe was arranged below the discharge circuit and steam made to escape very slowly, so as to fill the whole space between the needle points with dense fog. Against expectation it was found that a much higher voltage was required to strike across the same distance in fog as in dry air. This is seen in Fig. 16. Hence fog increases the dielectric strength of air and reduces the striking distance very markedly. This may be explained by the assumption that the discharge is by the fog broken up in an infinite number of minute successive discharges, between the conducting fog particles, requiring a higher total voltage than the direct discharge over the same total distance.

By turning full steam on, issuing from a boiler at 80 pounds pressure into the space between the needle points, the striking distance was still reduced further below that in fog, that is, live steam at atmospheric pressure has a much greater dielectric strength than dry air.

From these tests, the following results are derived:

1st. At constant voltage and constant wave shape, that is constant ratio between maximum and effective E. M. F., the striking distance is a constant, especially between sharp points, where the tests have been repeated over and over again, and independent of the atmospheric condition, the frequency, etc., to such an extent that the striking distance between needle points offers the most reliable means to determine very high voltages. For this reason, it is used in this manner as final check in all high potential insulation tests of the General Electric Company.

2nd. No physical law has been found to represent satisfactorily all the observations. Some point to the existence of a constant dielectric strength of air, analogous to the tensile strength of mechanics. Others point to the existence of a spurious counter E. M. F. of the spark or transition resistance from electrode to air.

3rd. Constant dielectric strength. Cylinders of 1.11″ diameter give an average disruptive strength of air of 60 kilovolts per inch. Cylinders of .315″ diameter, an average dielectric strength of 77. Spheres at very small distance point toward the latter value. As a disturbing factor in this case, enters the electrostatic brush discharge, which by a partial breakdown of the air surrounding the electrodes changes and increases the size and decreases the distance of the effective terminals.

4th. Counter E. M. F. of the sparks. The tests with sharp points give 22 kilovolts, or 11 kilovolts for a single transition from terminal to air. Spheres give curves pointing to a similar phenomenon. Electric conductors inserted at right angles into or parallel with the discharge, point to the existence of a counter E. M. F. of the same magnitude. The beginning of the electrostatic brush discharge is at a potential of this magnitude also.

5th. Potentials of 160,000 volts effective and even up to 170,000 volts have been experimented with. They are probably the highest voltages ever reached by man at ordinary frequencies with alternating currents of considerable power.

A paper to be presented at the 15th General Meeting of the American Institute of Electrical Engineers, Omaha, June 27th, 1898.

[ADVANCE COPY—SUBJECT TO REVISION.]

TWO WIRE DISTRIBUTING SYSTEMS AND LAMPS, 200—240 VOLTS.

BY JOHN W. HOWELL.

Distributing systems, operating at 200—240 volts with simple two-wire circuits, first attracted attention about four or five years ago. These early plants used two 110—120-volt lamps in series and were installed as a means of competing in cost of installation with three-wire plants without resorting to the alternating current system, with its disadvantages where motors are operated.

The growth of these plants was slow until the demand created by them led to the production of the 200—240-volt lamp and practical experience had shown the weaknesses of these lamps and led the makers to overcome the difficulties met in their manufacture.

What these weaknesses were, can be judged by the following extracts from previously published articles:

"The consensus of opinion at the present day of the average types of high-voltage lamps undoubtedly points to the fact that a large percentage are expected to short-circuit as soon as they are put up, and I have heard several engineers say that they expect about one in twelve to go in this way."

"There is no doubt whatever that almost all the present day 200-volt lamps are only suitable for burning in a vertical position. As soon as any other position is adopted defects become prominent. The long thin filament soon drops onto the bulb and cracks it. Also electrostatic attractions, owing to higher voltage, cannot be resisted by the long thin filament, and this is an additional cause of the filament approaching the bulb.

The effect of electrostatic attractions on long thin filaments is even noticeable with lamps burning in a vertical position."

These are extracts from a paper read by Mr. G. Bingswanger Byng before the British *Institution of Electrical Engineers* on February 24th, 1898.

These defects have been entirely remedied, and the experience of many plants operating 200—240-volt lamps during the past year proves these lamps to be just as reliable in service as are 110-volt lamps. The development of the lamp has taken considerable time, and the growth of the 200—240-volt 2-wire system in spite of the shortcoming of the early lamp is due to compensating advantages in other elements of the system.

These advantages over the three wire system are—(1) cheapness of installation and (2)—simplicity of operation.

1.—Cheapness of installation of the 200—240-volt 2-wire system, when compared with the 110—120-volt 3-wire system, arises from the use of a single large dynamo where the three-wire system requires two, each having half the capacity; with the accompanying duplication of regulating apparatus, indicators ammeters, etc. There is also a saving of the entire cost of the neutral conductor of the 3-wire system, which conductor in street mains (as distinguished from feeders) and in house wiring, should be as large as the outside wires.

Three-wire plants in cities, whose business warrants the employment of the skill necessary to operate the system to best advantage, successfully use 110—120-volt 3.1 watts per candle lamps and get very excellent results from them. The 200—240-volt 2-wire system cannot be considered a competitor of the 3-wire system under these conditions since the best 200—240-volt lamp on the market to-day is a four watts per candle lamp. But it is my opinion that in smaller installations, whose conditions call for a 3.5 watts per candle lamp when a 110—120-volt 3-wire system is used, the advantages of the 2-wire 200—240 volt system are sufficient to offset the disadvantages of a 4 as compared with a 3.4 watts per candle lamp. Under normal conditions the performance of the present four watts per candle 200—240-volt lamp as regards life and maintenance of candle power is about 25 per cent. poorer than the present 3.5 watts per candle—110—120-volt lamp, but it is my opinion that the greater irregularities in pressure ordinarily found in the 3-wire plants we are now considering, as compared with the 2-wire plants operated under similar condi-

tions, fully counteracts, by its injurious effects upon the lamps, the advantage in life and candle power which the 110—120-volt 3.5 watts per candle lamp has under normal conditions over the 200—240-volt four watts per candle lamp.

In considering the relative advantages of the two systems, in the case of moderate sized plants above referred to, we must charge against the saving effected in a 2-wire plant by its single equipment and omission of the neutral conductor, the increased capacity of machinery and conductors made necessary by a four watts per candle as compared with a 3.5 watts per candle lamp.

Considering the copper required by a 220-volt 2-wire system using four w. p. c. lamps as our unit, the copper required by the two outside conductors of a 110-volt 3-wire system using 3.5 w .p. c. lamps will be .875. The amount to be added to this for the neutral conductor will vary somewhat with location of the station with reference to the points of consumption. If we assume that one half the copper is installed for feeders and one half for the distributing mains, and that the neutral wire is one third the size of the outside wire in feeders and the same size in mains, then the total copper in the neutral conductors of feeders and mains will be 29 per cent. of the amount used in the 2-wire 220-volt system, and the total copper used in the 110-volt 3-wire system will be 16 per cent. greater than the copper used in the 220-volt 2-wire system. This comparison of the costs of copper does not consider the cost of wiring customers premises, which is usually borne by the customer; it is, however, an advantage to the 220-volt system that this wiring will require at least 20 per cent. more copper for the 3-wire 110-volt system than for the 220-volt 2-wire system.

The 4 w. p. c., as compared with the 3.5 w. p. c. lamp, would require for the 200—240-volt 2-wire system, one seventh greater capacity of boilers, engines and dynamos than would be required for the 3-wire 110—120-volt system.

The duplication of apparatus required for the 3-wire system will only partly offset this increased cost in the 2-wire 200—240-volt system, but the coast of this duplication of apparatus, together with the increased cost of copper required, will in many cases make the cost of installation of the 3-wire system at least equal to that of the 200—240-volt 2-wire system, leaving the simplicity of operation of the 2-wire system to be balanced against the increased cost of fuel required to operate 4 w. p. c., as compared with 3.5 w. p. c. lamps.

During the time the plant is operated considerably below its rated capacity, which will probably cover even the maximum load during the first year of the plant's existence, and all but three or four hours per day at all times, I think the increased fuel consumption due to 4 W. P. C. lamps will not be appreciable because of the poor efficiency of boilers and engines with light loads, and the total increased consumption of coal during the year will, I believe, be a small consideration under most circumstances, and in my opinion not enough to balance the simplicity of operation above noted.

A few months ago I made a trip through some of our central states, visiting a number of 200—240-volt plants to get a knowledge of the practical workings of the systems from the persons actually operating them, hoping to get from their experience, information concerning existing conditions which would be of value in the lamp factory.

Naturally, my first questions to the man who operated the plant concerned the lamps; were they generally satisfactory? were they long lived? did they ever explode? I was greatly pleased with the answers I received. The lamps were reported to be almost universally satisfactory. They were very long lived, in fact too long lived, for better service would be rendered if the lamps were renewed when they began to get dull, and not allowed to remain in service after they ceased to give a first class light.

These 4 W. P. C. 200—240-volt lamps may be used for 600 hours and still give a good light, but I think this is about the limit of their useful life. If not taken in they will keep on burning for a long time and will give an average life of over double that figure before breaking.

The reports concerning exploding were equally satisfactory. For the past year or more, exploding lamps had been practically unknown; previous to that time, however, this had been one of the most serious objections to the system.

All these plants have been installed with switches, cut-outs, and other appliances made for 110—120-volt installations, and it is remarkable how little trouble has been caused by them.

Sockets.

Ordinary key sockets have been freely used and experience has shown the Edison screw socket and lamp base to be preferable to other types. Some engineers recommend keyless sockets and good snap switches, to avoid possible failure of the key socket to break the circuit.

In order to get the desired factor of safety, the General Electric Company has recently developed a special key socket for this work, having a break gap twice as great as the sockets designed for 110—120-volt systems, and having a fibre insulating lining between the outside shell of the socket and the screw shell which is connected with the circuit. This fibre lining projects sufficiently beyond the screw shell to prevent contact with the circuit when the lamp is in the socket. The lamps are provided with short bases designed to be completely covered by this fibre lining, so it is impossible to get a shock by touching the lamp when it is burning.

Flexible Cord:

Flexible cord has been freely used, and is all right if the best quality is used. For this purpose the conductors should be covered with rubber $\frac{1}{32}''$ thick and the outside covering should be silk, or if cotton it should be slicked with a compound which makes it non-inflammable.

Rosettes:

Rosettes designed for 110—120-volt circuits having fuses, have given a good deal of trouble, so it has become good practice to leave out the fuse wire and put copper wire in its place thus having no fuse between the lamp and the fuse block close to the switch controlling the circuit.

The General Electric Company has produced a new porcelain rosette which has fuse wires over one inch long, these have been severely tested with very satisfactory results. Their use with fuses will give protection to individual lamps, which is very desirable, as it increases the safety of the wiring.

Cut-outs:

Fusible cut-outs designed for 110—120-volts, have failed in many instances, and have often proved much more fusible than their makers expected them to be.

A complete line of fuse blocks carrying special 200—240-volt fuses has been developed recently by the General Electric Company. These are fitted with the cartridge type of fuse, the fuse being contained in a tube which is provided with brass terminals which fit in spring clips on the fuse block. The fuse wire is inside the tube and is soldered to the brass terminals. The tube is then filled with a plaster compound which smothers the arc

formed when the fuse blows. These fuses have been severely tested with 500 volts and currents many times their rated capacities, and have always opened the circuits without a flash.

SWITCHES:

Standard knife switches or quick-acting snap-switches having large break distances, should be used. Such switches specially designed for 200—240-volt installations are now being developed.

WIRING:

Standard methods of wiring for 110—120-volts are approved for 220—240-volts. Good rubber covered wire run on porcelain knobs or cleats is specified by the fire underwriters.

ARC LAMPS:

Good arc lamps burning two in series on 220-volt circuits are on the market. Each lamp is provided with an automatic cut-out which throws an equivalent resistance into the circuit when one lamp is extinguished, thus permitting the use of one lamp if desired.

FUTURE:

The future of the 200—240-volt system depends largely upon the development of a more economical lamp. All other elements of the system can be made entirely satisfactory with the means and knowledge now at our command, but the production of a more economical lamp calls for an advance in the art of lamp manufacture. That this advance will be made and a more economical lamp will be produced, I have no doubt, and hope to see such a lamp placed upon the market at an early date.

A Paper to be presented at the 15th General Meeting of the American Institute of Electrical Engineers, Omaha, June 27th, 1898.

[ADVANCE COPY—SUBJECT TO REVISION.]

A CAPILLARY ELECTROMETER FOR ELECTRICAL MEASUREMENTS.

BY CHARLES FREDERICK BURGESS.

Many of the measurements made in practical engineering work require the use of a sensitive electrical instrument, and owing to the demand for such an instrument the galvanometer has reached a high degree of perfection in some of the latest and improved forms. There is still much to be desired, however, in the way of portability, rapidity of reading, simplicity, cost, and freedom from the effects of external influences such as mechanical vibration and varying magnetic field. There can scarcely be a call for an instrument of greater sensitiveness and accuracy than that which the galvanometer gives us when properly constructed and used, and in fact this sensitiveness might well be reduced in many cases if by so doing some of the disadvantages in its use might be overcome.

It is my purpose to call attention to the applicability of the capillary electrometer for practical electrical measurements, as a means of overcoming some of these defects. Although the principle upon which this instrument works was discovered about twenty years ago, its use has hardly extended beyond a very limited application in scientific research, and from some of the excellent properties which it holds as a measuring instrument, this seems to be a surprising fact.

This principle is the change of surface tension of two liquids which are in contact (preferably mercury and dilute sulphuric acid) in a capillary tube, and the meniscus which is formed at the separating surface moves, upon the application of a potential difference to the two liquids in question. It is the movement of

this meniscus which corresponds to the movement of the galvanometer needle, in using it as a measuring instrument. I do not propose to enter upon a scientific discussion of the phenomena of surface tension, capillarity, and the change of surface tension under the influence of electrical potential which are here utilized, as this has been completely done in a number of scientific publications,[1] but will give a description of a certain form of this instrument and enumerate some of its principal characteristics.

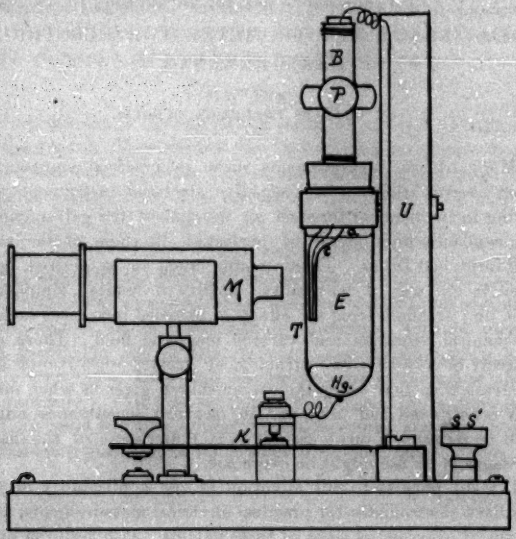

FIG. 1.

The only reasons for the comparatively limited use to which the capillary electrometer has been put, appear to be that it has not been made into a portable form, as glass and two liquids enter into its construction, and that great care is necessary to keep it in good condition. The form of instrument herein described was designed with the view of overcoming these objec-

1. The Capillary Electrometer, George J. Burch, Lond. *Electrician*, '96, "Lehrbuch der Allgemeinen Chemie."—Oswald, Vol. II., p. 485.

tions. Fig. 1 illustrates the construction, T being a rather thick glass tube having a platinum wire sealed in the lower end. A layer of mercury covers the platinum wire, and the tube is nearly filled with the electrolyte E, which consists preferably of a dilute solution of sulphuric acid. A capillary tube C, made by drawing out a piece of thermometer tubing, rests against the side of the glass, and is held firmly in place by a tightly fitting rubber stopper inserted in the top of the large glass tube, enclosing an air space A above the electrolyte. A piece of flexible rubber tubing is fitted on the upper end of the capillary and wrapped tightly to prevent leakage. This tube, being filled with pure mercury is closed by a piece of glass into which platinum wire is sealed, and again wrapped tightly with copper wire. On compressing the pinch-cock P, mercury is forced through the capillary and upon slightly releasing the pressure the mercury is drawn back, being followed by the electrolyte. The instrument is then ready for use. A microscope M containing a micrometer eye-piece is held on an adjustable support, and is used for noting the movement of the meniscus. The glass tube is held firmly by a brass collar attached to an ebonite upright. Two binding posts, S, S, form the terminals of the instrument, K being the key which short-circuits it when not in use and connects the electrometer with the binding posts when depressed. The connecting wires are placed in grooves between the two parts of the base.

This instrument is one which may be tipped in any position without injury, it is as portable as any instrument made partly of glass can be, and it may readily be taken apart for the purpose of cleaning or renewing its parts. It will respond to a difference of potential of one one-thousandth of a volt, though with proper attention to its construction and by use of a suitable microscope its sensibility may be increased ten-fold or even greater. If a pressure over a volt is applied to the terminals an electrolytic action will occur, with the liberation of a bubble of gas or the formation of a cloudy precipitate at the meniscus, depending upon the polarity. This will destroy the sensibility, but the defect can be easily remedied by compressing the rubber tube until a drop of mercury flows through, and on releasing the pressure the instrument is again ready for use. The adjustment of the meniscus on the micrometer scale may be made either by moving the microscope or by means of the pinch-cock.

The direction of movement of the meniscus depends upon the polarity, and the extent is approximately proportional, through a limited range, to the potential. This is not absolutely so, for it would entail the use of a capillary of absolutely uniform bore, which would be difficult to attain. For this reason it is almost essential to use it as a zero instrument.

The meniscus moves, not by reason of a passage of current through the instrument, but by the application of a potential difference which gives it a certain electrostatic charge. In other words, it may be said to be a condenser which automatically registers the amount of charge which it holds. An addition of resistance in circuit will in no wise decrease its degree of sensitiveness, though it will make the movement of the meniscus somewhat slower. With the instrument described, an application of a pressure of .005 volt produced a deflection of eight divisions on the scale whether this pressure was applied direct or through a resistance of 100,000 ohms.

On pressing the electrometer key the meniscus will move very rapidly if a potential difference exists at terminals, and again on closing the short-circuit key it will quickly return to its zero point without vibration. It may be subjected to almost any amount of mechanical vibration without moving the position of the meniscus on the scale.

The principal characteristics may be summed up as follows: The instrument is of very simple construction and can be made by any one fairly expert in glass manipulation. It is very light and compact, and requires little space for its operation. The instrument described weighs two pounds and occupies a space of 7″ x 5″ x 6″. In the way of portability little more can be desired. It may be quickly set up for use, requiring no leveling, and the adjustment may be rapidly made. No serious damage such as a burnout can occur by the accidental application of even a very high pressure. The breakage of any of its parts can be repaired with little trouble and at slight expense. One of these instruments has been in use for nearly a year and the only repairs necessary on it was in once refilling the rubber tube with mercury. The entire absence of the effect of mechanical vibration and varying magnetic field will especially recommend this instrument. On the other hand, it can hardly be made with the degree of sensitiveness which may be given the galvanometer, and it has to be used as a zero instrument.

Measurement of Insulation Resistance.—Having had occasion to measure the insulation resistance of a large number of samples of rubber covered wires in lengths of 500 feet, I attempted to make the measurements with a four-coil Thomson astatic galvanometer, using the well-known direct deflection method, and encountered all the difficulties which are attendant upon the measurement of such high resistances, and was unable to obtain satisfactory results, owing to the mechanical vibrations in the building in which the work was to be done, and to the constantly changing magnetic field produced by electric machinery operating in the room below. When trouble was not experienced from this last source, the movement of cars on an electric railway several hundred yards away caused a decided swing of the galvanometer needle. An improved and sensitive form of D'Arsonval galvanometer was then used, by means of which these influences were greatly reduced, though not entirely eliminated. The tendency of the suspension wire to take a set if the swing of the coil was somewhat large, made a frequent determination of zero necessary, and this, together with the length of time required for the mirror to come to rest made the measurement of a large number of resistances slow and tedious work.

To use a battery of chloride of silver cells for such measurements may well be objected to for testing insulated conductors, and especially those which are to be used on high-tension working, on account of the comparatively low electromotive force which is necessarily used, and the internal resistance of the cells which is a greatly varying quantity and may sometimes reach an enormous value. It is desirable in such measurements that the wire shall be tested under actual working conditions, and a pressure even greater than that for which it was designed is preferable. For this reason, a one K. W. dynamo to be operated at a pressure of 1,000 volts was obtained. This was belt-driven from a motor, and it operated at a fairly constant pressure, but upon attempting to employ the galvanometer for the measurement of the current flowing through the insulation, it was found that even the slight variations of pressure, combined with the electrostatic capacity of the cable under test, caused disturbing currents to flow through the galvanometer, thus preventing the needle from taking a steady deflection.

The attempt was then made to employ the capillary electrometer in place of the galvanometer, and the results obtained

were so satisfactory that I feel justified in describing the method used and the results obtained. On account of the electrometer being a zero instrument, the direct deflection method cannot be used, and a compensation method is employed; the method of operating being as follows. Fig. 2 shows arrangement of connections for the test.

A known resistance R (preferably 1,000,000 ohms) is placed in series with the generator and the insulation to be measured. The electronometer C is employed in measuring the fall of potential across R by the compensation method of introducing a pressure in series with the electrometer which shall just neutralize the pressure across R. When this has been effected, the

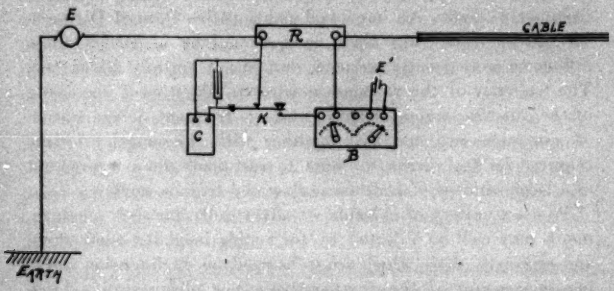

FIG. 2.

depression of the electronometer key will produce no movement of the meniscus. It is therefore necessary to have an electromotive force which may be varied at will. A convenient method of obtaining this pressure is the following:

Fig. 3 represents plan of resistance box, the terminals of which are binding posts A, B, C, D, E. Between D and E are connected coils of German silver wire arranged in series, and giving a resistance of 1,000 ohms. This resistance is subdivided into nine 100-ohm coils and ten 10-ohm coils, the ends of which are connected to brass terminals and numbered as shown. Two brass arms which are connected with wires to binding posts A and B make contact with the resistance terminals. It is by noting the position of these arms that the pressure between A and B is determined, that pressure being the resistance included between

the two arms X a constant pressure at terminals D E, ÷ 1,000. When the balancing pressure requires that the contact be made between the terminals of one of the ten-ohm coils, the resistance which would produce an exact balance, may be calculated by interpolation with a fraction of an ohm, this being done by noting the distance of movement of the meniscus of electrometer. A 9,000-ohm coil between C and D enables the pressure across D E to be one-tenth of the pressure of the battery or other sources of constant pressure connected to C and E.

The data necessary to derive the value of insulation resistance then is the value of the compensating E. M. F. which may be obtained directly from the reading of the box B when a balance

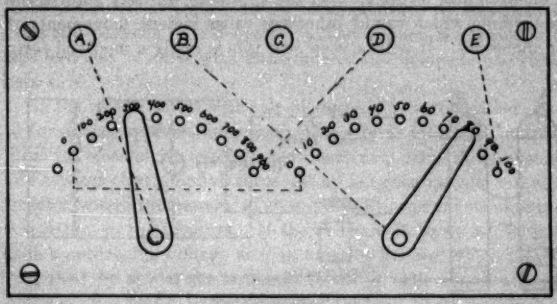

FIG. 8.

has been effected. Any ordinary form of primary or secondary cell may be used for pressure E'. The expression for insulation resistance is

$$\text{Resistance} = \frac{E \times 1{,}000{,}000{,}000}{E' \times B} - 1{,}000{,}000.$$

The constant is 1,000,000,000 E/E', and the ratio E/E' may be derived by means of a voltmeter, or if such instrument is not available, by balancing the smaller pressure in circuit with the electrometer and terminal A B against the larger pressure connected to C E, the volume being 1,000 ÷ box reading.

When dynamo pressure is used for testing, the variations in current flowing through R caused by slight but rapid fluctuations of applied pressure and the electrostatic capacity of the cable, will produce a very rapid movement of the meniscus which will

interfere seriously with its use unless suitable precautions are taken. The influence of this variable current upon the electrometer may be reduced to an unnoticeable amount by placing a condenser across the terminals of the electrometer. To effect this, however, a comparatively large condenser capacity is necessary, ten microfarads being used in the apparatus which I employed. The following arrangement proved much more satisfactory however than the ordinary form of condenser first used.

Fig. 4 shows an H tube of glass closed at the lower ends in which platinum wires are fused. These wires are covered with mercury and the tubes then nearly filled with an electrolyte, very dilute sulphuric acid being suitable. The platinum wires are connected to electrometer terminals and the appliance entirely absorbs the fluctuations which would otherwise cause violent movements of the meniscus. It will be seen that this is really a modified form

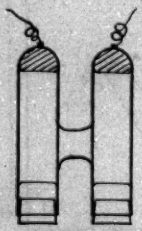

FIG. 4.

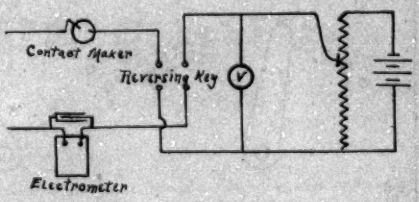

FIG. 5.

of the electrometer in which the surfaces of contact of electrolyte with mercury is rather large. Using tubes .5 in. in diameter a condenser of 100 microfarads capacity may readily be obtained. If this capacity is too great however, the movement of the meniscus in the capillary tube may be so slow as to consume considerable time in waiting for it to come to rest. The capacity may be varied by varying the amount of mercury surface exposed, or by changing the density of the electrolyte.

By such an arrangement the pressure from any ordinary direct current dynamo running at a reasonably constant voltage may be utilized in the testing of insulation resistances.

Values for insulation resistance may be obtained with considerable accuracy and speed, and values as high as 20,000 megohms have been measured. The limit may be made very much greater by increasing the sensitiveness of the electrometer or by increasing the value of R.

A portable testing set which I have been using for some time consists of the following parts:—Chloride of silver testing battery of about 50 volts; capillary electrometer with microscope and short-circuit key; graphite resistance of one megohm; thousand ohm subdivided resistance box; a 1.5-volt dry cell. The weight of set, exclusive of 50-volt battery is six lbs., the chloride of silver cells weighing 10.5 lbs. The range is from 0 to 5,000 megohms.

Measurement of Electromotive Force.—The electrometer with subdivided resistance box may be used for the measurement of direct pressures up to 150 volts with an accuracy of one-tenth of one per cent., provided resistance is accurate. The standard for comparison is the standard cell of known E. M. F. The pressure to be measured is placed across terminals D. E. if less than 15 volts and between C. E. if greater. The balance is effected with standard cell in series with electrometer and connected to terminals A. B.

As the cell in series with the electrometer is not called upon to furnish an appreciable current, this affords an excellent apparatus for electrolytic measurements such as polarization pressures and contact electromotive force set up between metals and electrolytes, where the passage of an appreciable current would cause a variation in the pressure. If the pressure to be measured is of such a nature, the voltage at D. E. must be known, which may be furnished by any of the ordinary forms of cells which will not polarize appreciably when connected through 1000 ohms.

Values of current may be obtained by the use of standard resistances of large current carrying capacity, the drop through such standard being measured by connecting it in electrometer circuit.

The calibration of direct current ammeters and voltmeters may be conveniently and rapidly carried out with the above arrangement.

This form of electrometer is especially suited for use in the dynamo room or in the neighborhood of moving machinery, on account of the small amount of space necessary for its operation, the rapidity with which measurements may be taken, and the entire freedom from external influences. It has been used to a considerable extent in the laboratories of the University of Wisconsin in obtaining current and pressure curves on alternating current machinery, and has proved satisfactory where other

forms of measuring instruments had failed. The method of using this instrument for tracing alternating current curves is in substituting it for the galvanometer or the telephone receiver in the method proposed by Mershon[1], Fig. 5. It is superior to the telephone receiver from the fact that its readings depend upon the eye rather than the ear; the direction in which the slider must be moved to effect the balance can be detected immediately by noting the direction of movement of the meniscus; in addition, the telephone receiver is difficult to work with on account of the noise of belts and machinery and the induction which is set up when used in the neighborhood of alternating current apparatus. The degree of accuracy is considerably greater as is also the rapidity with which curves can be taken. For use in this work a microscope is unnecessary, and when the movement of the meniscus is too rapid and of a fluctuating nature due to unsteadiness of the compensating pressure or other causes, a condenser similar to the one described above may be used as a damper. A resistance placed in series with the electrometer will also serve to increase the period of movement of the meniscus.

The above are some of the measurements in which the use of the electrometer has been tested, and the many advantages which have been found for it leads me to believe that it might be advantageously used in many other measurements in practical engineering work, especially in those where a great degree of sensitiveness is not required. In any of the modified forms of Wheatstone bridge methods for the measurement of resistance, induction and capacity it should prove a serviceable instrument. For use on shipboard it would be the ideal instrument as far as the rolling, and the jars due to moving machinery are concerned.

Another property of this instrument, and one which might be utilized to advantage, is the great rapidity with which the meniscus may move under a variation of pressure. It has been shown that alternating curves of a frequency of 120 per second may be obtained by a photographic record of the excursions of the meniscus, as well as the currents of much greater frequency which the telephone line transmits.

1. "Alternating Currents and Alt. Current Machinery," by D. C. and J. P. Jackson, p. 597.

A paper to be presented at the 15th General Meeting of the American Institute of Electrical Engineers, Omaha, June 20th, 1898.

[ADVANCE COPY—SUBJECT TO REVISION.]

ALTERNATING CURRENT TRANSFORMERS—FROM THE STATION MANAGER'S VIEWPOINT.

BY W. F. WHITE.

In the early days of alternating central station development, many a sale of alternating central station equipment was lost to direct current competitors by the facts, figures and fancies which the salesman of direct current apparatus was able to advance concerning the wastefulness of energy, the expense of maintenance, and the danger to life which purchasers must accept as inseparably wedded to any transformer system.

Notwithstanding that the danger to life was unfortunately made much of, and the facts grossly misrepresented, still the correctness of many of those arguments, in the light of present knowledge, stand up remarkably well under the scrutiny of qualitative analysis. And it is even more remarkable, in view of the natural tendency to exaggerate induced by zeal to make sales, how little, if any, were the facts of these evils overdrawn or magnified. Quantitative analysis in fact proves that most figures then given to show the amount of transformer "leakages" were understated. On the part of salesmen this was undoubtedly more a mistake of the head than of the heart, for while some few of them knew the results of some isolated tests of some particular size of one type or manufacture of transformer, no one, whether salesman, engineer and designer, or manufacturer, had any knowledge or conception of the cumulative effect of transformer losses on the station output. Such little knowledge as had then been acquired related more to the transformer efficiency under certain percentages of rated load, than to all day efficiency, or the 24-hour ratio of energy output to energy absorbed under average conditions.

Little, if anything, was known of transformer core losses, and the effect of power factor on station and line capacities had not had consideration. As long ago as 1892 the writer, in making a careful examination into the operations of one of the then largest alternating stations in this country, made the discovery that with a net meter rate to consumers of about 20 cents per 1,000 watt hours, the income per K. W. hour output at the switchboard was only about six cents, showing an average loss from station switchboard to consumers' meter of about 70 per cent.

Current was supplied, in accordance with the then existing universal practice, through one or more transformers for each customer. In two or three cases, for large installations, as many as 18 or 20 transformers were erected for a single customer, no transformer of more than 75 lights capacity being used; the transformer primaries connected in multiple, but the secondaries feeding entirely distinct and separate circuits.

Of each 1,000 watt-hours output at the switchboard, it was estimated that approximately 300 watt-hours were delivered through the customers' secondary meter, 600 watt-hours were accounted for by transformer core losses, leaving 100 watt-hours to be accounted for by transmission and other losses.

A statement of these conclusions to a well-known manufacturer, who supplied the wattmeters and many of the transformers, elicited the opinion that either the primary meters registered too fast or the secondary meters too slow.

A meter expert from the factory accordingly spent several weeks in testing all primary and secondary meters, found them correct, and proved that the apparent discrepancy was not due to the meter end of the plant equipment. With the writer's recommendation that a three-wire secondary network be erected, using 100-volt lamps, and fed through large transformers at frequent centers of distribution, the matter was dropped; but there is no doubt that the proportion of the output attributed to transformer losses was approximately correct.

Since that time it has been the writer's good fortune to be in touch with a considerable number of alternating plants, giving him knowledge of their operating results, and he does not hesitate to assert that very few plants exist, giving continuous service, using separate transformers for individual customers, and charging from twelve to twenty cents per 1,000 watt hours, secondary meter measurement, whose income per

K. W. hour output at the switchboard exceeds from four cents to seven cents. In other words, the transmission, transforming and distribution losses are commonly as much as two hundred per cent. of the customers' demand, as shown by the customers' meters. This does not mean operating expenses three times as great as if all these losses were eliminated, but it does mean that large expenditures are made for fuel and feed water for the keeping warm of numerous transformers operating about 20 hours per day with open secondaries. And great numbers of small transformers also mean great variations of secondary voltage, large bills for lamp renewals, unsatisfactory service, and a reduced number of lamps which can be served at any instant from a given generator capacity. To show the benefits to be derived by using a secondary net work, fed by large transformers working in multiple, but located one at each feeding point or local center of distribution, as compared with the use of small transformers, one or more to a customer, the writer presents some results accomplished by an existing plant in remodeling its transformer system in accordance with these ideas.

January 1st, 1895, this plant was serving 16,702 incandescent lamps (16 c. P. equivalent) through 547 transformers of 22,190 lamps rated capacity, each transformer averaging 40.6 lamps capacity and serving 30.6 lamps. The transformer load, with open secondaries, was, as near as could be determined, about 80 amperes at 1,000 volts. In view of the fact that the maximum load never exceeded 50 per cent. of the lamp capacity connected, it was evident that a very considerable reduction could be made both in the number of transformers connected and in their aggregate capacity, by the use of a secondary network. The theoretical limit was to reduce the transformer capacity to equal the maximum load, which would lower the capacity connected from 22,190 lamps to about 7,000 lamps, a reduction of about 64 per cent. This limit was evidently impossible of attainment because of the large number of isolated installations, especially in the residence districts, requiring individual transformers, which of necessity must generally be of capacity equal to several times the average load. A system of 3-wire secondary networks was laid out, and estimates made showing a possible reduction of almost 80 per cent. in the number, and of about 50 per cent. in the lamp capacity, of transformers connected. A large number of tests of core losses of all sizes of transformers in service was

then made, for estimating the total core losses for the plant, and for comparisons with the estimated core losses of the large transformers to be erected in replacing the old.

An illustration of one or two of these comparisons may be of interest. One installation of approximately 700 lamps was fed through a large bank of small transformers having an aggregate core loss of 4,440 watts, which could be replaced by two transformers having a combined estimated core loss of 310 watts. On the basis of the number of watt hours per pound of coal then being produced by the plant, not taking into account any line losses, this example shows the following results:—

4,440 watts,	38,894,400 watt-hours per year—requires	190.6 tons coal
310 "	2,715,600 " " " "	8.4 " "
Annual saving,	36,178,800 watt-hours	182.2 tons coal

The annual income from this installation was about $2,100.00, at twelve cents per 1,000 watt-hours. With a secondary output of about 17,800,000 watt-hours, and transformer core loss of about 38,894,400 watt-hours, the income per K. W. hour output at the switchboard for this installation was evidently less than 4 cents, and the core losses were more than double the total secondary output.

In another case, one section of the city, comprising 10 city blocks, was fed by one dynamo and circuit through 71 transformers, having an aggregate rated capacity of 4,482 lights, and aggregate core losses of 10,402 watts (estimated), or 91,121,520 watt hours per year.

To replace these transformers, a 3-wire secondary network was erected, supplying all customers, and fed by five 400-light transformers, located at as many feeding points or local centers of distribution. Each transformer was provided with primary and secondary switches, by means of which each night all transformers were thrown into service from dusk until midnight, at which latter hour four transformers were cut absolutely out of circuit. On a yearly average, five transformers were in circuit about six hours per day, and one transformer the remaining 18 hours per day. The combined core losses of the five were about 1,250 watts, and for the hours in service aggregated about 4,380,000 watt hours per year. Some results of the above changes on this one circuit therefore were as follows:

Number of Transformers.	Lights Capacity.	Total Core Losses—Watts.	Total Annual Core Losses—Watt Hours.	Equivalent Tons Coal.
71	4,482	10,402	91,121,520	446.7
5	2,000	1,250	4,380,000	21.5
Saving.	2,482	9,152	86,741,520	425.2

A reduction of over 95 per cent. in core losses, and a saving for one year in cost of fuel alone of $815.40 were effected for this one circuit. These examples were extremes, because the lighting covered by them was concentrated; and because in most residence and outlying business districts few changes in the number and capacity of transformers could be made. The results shown by the last example were therefore much above the average for the whole plant, but serve to show the possibilities of such changes and the methods pursued in this case.

Some results for the plant as a whole may therefore be fairly indicative of what other average plants operating under similar conditions may accomplish in the same direction, and were as follows:

Time.	Lamps Connected.	Transformers Connected.	Total Capacity.	Average Lights Capacity.	Average Lights Served.
Jan. 1, 1895.	16,702	547	22,190	40.6	30.6
" 1, 1896.	19,989	201	13,865	69.	90.1
" 1, 1897.	20,262	152	13,498	88.8	133.3
" 1, 1898.	23,643	141	14,946	106.	167.7

Year.	Income.	Switchboard Output, K. W. Hours.	Income per K. W. Hour Output.	Increase of Income per K. W. Hour Output.
1894	$59,012.71	928,400	$.0635	
1895	54,231.67	791,732	.0684	7.7 per cent.
1896	51,415.09	551,064	.0933	46.9 per cent.
1897	56,317.42	508,070	.1108	74.5 per cent.

Year.	Actual K. W. Hours Output.	Estimated K. W. Hours Output—Basis 1894. Conditions.	Estimated K. W. Hours saved by Change.	Estimated Saving in Coal—1894 Basis.	
				Tons.	Cost.
1894	928,400				
1895	791,752	854,042	63,290	310.2	$670.91
1896	551,664	809,689	258,625	1,267.8	2,641.42
1897	508,070	886,888	378,818	1,856.9	4,015 43

It must be borne in mind that the number of 16 C. P. equivalents connected Jan. 1, 1895, was 16,702, and increased by Jan. 1, 1898, to 23,643, or more than 41.5 per cent. The savings estimated in last table are based on the conditions at the beginning of this period, *i. e.*, the same number of transformers and the same transformer capacity. But if the same conditions had obtained throughout, an increase of 41.5 per cent., or 226 transformers, of 9,208 lights capacity, would have been added to supply the new lights connected. And while only a part of the core losses of the transformers in service Jan. 1, 1895, was saved by the change, the entire losses of the 227 transformers, which would have been required by increased business, were saved. It is therefore perfectly safe to add 50 per cent. to the savings shown in the last table, in estimating actual benefits. From reduced core losses alone, therefore, there is an annual saving of approximately $6,000.00.

In core losses alone, the savings already effected have more than paid all costs of the entire change of lamps, meters and and transformers (for because of a simultaneous change from 52 volts to 106 volts, secondary, all lamps and meters also had to be changed), and of the erection of secondary network; and the annual saving of $6,000.00 is equal to 6 per cent. interest on $100,000.00, secured without one dollar invested.

To be certain that the saving of fuel was in proportion to the reduction in K. W. hours output, as estimated, the plant efficiency under the changed conditions must be known. It is therefore pleasing to note that, using coal from the same mine most of the time, the watt hours output per pound of coal actually increased for 1896 by 3.8 per cent., and for 1897 by 5.9 per cent. over 1895, so that the saving in fuel was at least not less than in proportion to the saving in K. W. hours output.

Some incidental advantages of the change may be mentioned.

First. Under the old plan, the 6,941 lights added would have required about 9,200 lights additional transformer capacity, of 40 lights average size, of $1.00 per light average cost, giving an investment saving of $9,200.00.

Second. The transformer load, with open secondaries, of 80 amperes, was reduced, by the fall of 1897, to about 30 amperes, notwithstanding an increase of about 25 per cent. in the number of lights connected. A saving of 50 amperes at 1,000 volts represents a capacity to serve, without additional station or line equipment, approximately 1,000-16 C. P. equivalents, all burning at one time, or fully 2,000-16 C. P. equivalents connected.

Third. Formerly as many as a dozen transformers were frequently burned out in a single thunderstorm. Ten per cent. of the transformers connected would be a conservative estimate of the number burned out each year from various causes. Of the new transformers connected during the past three years only one has failed from any cause, and that one was defective, and burned out under light load almost immediately upon being put in service. Transformer repair costs are therefore practically eliminated.

Fourth. Incandescent lamp renewals have been greatly reduced. Using lamps from the same factory, of the same efficiency, and bought at the same price per lamp, the cost (estimated) of renewals per K. W. hour consumed by the lamps has been reduced from $.0081 in 1895 to $.0057 in 1897, a reduction of 29.6 per cent. This saving represents the advantage of regulating the secondary voltage by pressure wires connected to the secondary network, as compared with the former method of regulating the primary voltage by station transformers.

Fifth. The principal remaining incidental advantage is the uniformity of voltage, bringing uniformity in quality of service and the consequent satisfaction of customers, the value of which cannot be estimated in dollars or per centages. These are some actual results, and such as any plant operating under similar conditions can achieve at small expense, if the changes are well planned, and the replaced apparatus sold to the best advantage.

In this connection, attention must be called to one other fact, which is that central stations using separate transformers for individual customers have the great majority of their transformers working on open secondaries probably 80 per cent. of the time. The best transformers have a power factor of about 50

with open secondaries, and of fully 99.9 with almost any load they carry in practice. It is therefore perfectly credible, and the writer has demonstrated it to be so by many actual tests on various plants, that the average power factor of plants operating under above specified conditions is approximately 65.

It scarcely needs demonstration that if, instead, a secondary network is used, and only such transformers allowed to remain in circuit at periods of light load as are required to advantageously carry such load, the average power factor will be approximately 100. The volt-ampere hours therefore, calculated from the station voltmeter and ammeter readings, give approximately the watt-hours output. But almost all existing plants use separate transformers for individual customers, and the great majority of them do not have primary wattmeters on their switchboards, and do calculate their volt-ampere-hours output as equivalent to watt hours, getting results about 50 per cent. in excess of the facts. They therefore are able, apparently, to show 50 per cent. more watt-hours per pound of coal, and 33⅓ per cent. less expense per K. W. hour output than another station operating under the same conditions and fully equalling theirs in actual performance, but which accepts the primary meter readings as the basis of estimate.

As a matter of fact, the usual disparity is greater than the percentages given, because most station managers in erecting primary switchboard meters, follow their aesthetic inclinations, and use ornamental glass covers which are particularly susceptible to the admission of dust and grit, with the result that the meters rapidly slow down. It is highly probable therefore that in some stations the volt-ampere-hour record would show twice the number of apparent watt-hours per pound of coal, and one-half the expense per apparent K. W. hour output that would be shown by the record of the primary meter already erected on the switchboard, which tends only to show the worthlessness of data from different plants, for purposes of comparison, unless the local conditions are known in each case.

This digression is permissible, we hope, because the average power factor, so far as we know, has never been taken into account, and because for that reason much misleading alternating central station data is in circulation.

Purchasers and users of transformers should insist that transformers meet certain requirements, or be rejected, and some of these requirements will be stated.

Where transformers are used for feeding networks, the secondaries in multiple, small core losses are of much greater importance than close regulation, and should be kept at the lowest practicable point. The regulation will be cared for by treating the transformer and feeder drop together, through the use of secondary pressure wires.

Where the transformers are entirely independent, feeding separate secondaries, close regulation must be had and the core losses be made as low as the adopted regulation will permit. With constant full load the temperature must not rise to a high point, and should not exceed 50° centigrade above the surrounding air. This is important not only because of the increased life of transformer insulation through low temperatures, but also, and possibly to a greater degree, because magnetic fatigue is a certain follower of high core temperatures. If therefore the core losses might be increased 50 per cent. or possibly 100 per cent. through high temperatures, the cost of these core losses, as shown in case of the plant above, make argument unnecessary as to the desirability of having transformers run cool.

The transformer case should be filled with oil, because of its insulating qualities, its exclusion of air, and because of its assistance in maintaining the low temperature desired.

The insulation resistance between the primary and secondary windings should be able to stand up under not less than 10,000 volts, alternating, difference of potential, for transformers having 1,000-volt or 2,000-volt primaries, and the writer would strongly urge a 15,000-volt, alternating, test. If both the primary and secondary windings are on the core, the same insulation resistance should be required between either winding and the core, as between the two windings. The use of grounded shields placed between the primary and the secondary windings, the grounding of the shell or core, the grounding of the secondaries, the placing in each customer's service connections of any device to automatically open the circuit of such customer in case of an unusual difference of potential, are all practices that are opposed as expensive, as liable to cause, more than to prevent trouble, as complicating existing practice, and as wholly unnecessary.

Proper insulation is sufficient protection against the damages sought to be avoided. Station managers as earnestly desire complete safety for life and property as the Underwriters' Association, but vigorously oppose the introduction of any unnecessary

devices that are expensive to install and to maintain. That Association, in seeking protection for the secondaries, should do so through sufficient transformer insulation and low temperatures. Both of these requirements mean increased cost of production and higher selling prices, but as the consequent increased economy makes the increased cost a profitable investment, no reasonable manager will object. In this way the ends sought will be best attained, and the interests of all parties duly safeguarded.

Omaha, Neb., May 19, 1898.

A paper to be presented at the 15th General Meeting of the American Institute of Electrical Engineers, Omaha, June 28th, 1898.

[ADVANCE COPY—SUBJECT TO REVISION.]

SOME PHASES OF THE RAPID TRANSIT PROBLEM.

BY ALBERT H. ARMSTRONG.

The rapid transit problem, in congested districts, has to deal with the transportation of passengers at a high average speed with frequent stops, and even in suburban traffic where the stops are less frequent, the schedule speeds have been so increased by the ability of the electric motor to accelerate rapidly, that trains hardly reach full speed before it is necessary to apply the brakes. As running at a constant speed does not occur in rapid transit service where stops are at all frequent, it becomes of the greatest importance to carefully investigate the subject of train acceleration in order to determine the method of running a train from station to station with the least expenditure of energy.

Problems in train acceleration may be divided into two broad classes: 1.—Where the road is level and

2.—Where grades exist or where an artificial profile is made in order to take advantage of down grades at starting.

Modern passenger cars demand a dead weight of approximately 550 lbs. for each passenger carried, or, in other words, only 20 per cent. of the total weight of a loaded motor car is a paying load. When it is considered that in rapid transit service with frequent stops, over 80 per cent. of the total energy output of the motive power on level roads is required to accelerate the train, it is evident that in this class of service the rolling stock requires the greatest attention.

To avoid undue complication, the following discussion has assumed that train friction is a constant quantity at all speeds, as at the low maximum speeds—20 to 30 miles per hour— reached

in practice with frequent stops of two or more per mile, the error introduced by assuming a constant friction rate will be small and will not at all alter the conclusions arrived at.

The following constants have been assumed as representing average operating conditions:

Length of run, 2000 feet.

Length of time train is in motion, 75 seconds.

Schedule speed, 16.05 miles per hour, including 10-second stops, or 85 seconds total time.

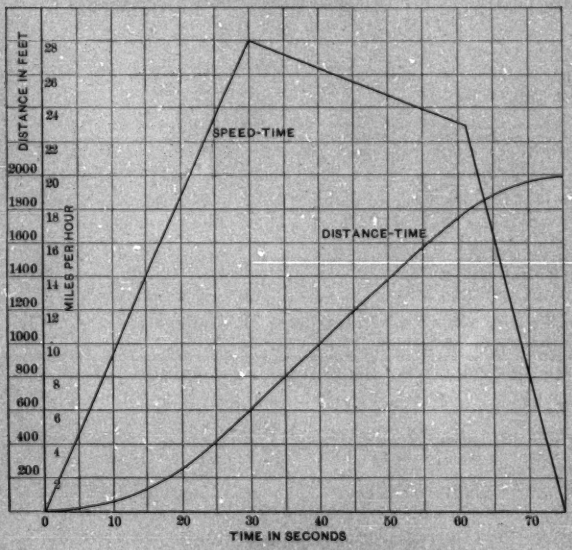

FIG. 1.—Acceleration Curves.
Distance, 2,000 feet. Time, 75 seconds. Tractive Effort, 100 lbs. per ton. Friction, 15 lbs. per ton. Braking Effort, 150 lbs. per ton.

Tractive effort, 100 lbs. per ton total, to be maintained uniform during acceleration of the train.

Braking effort, 150 lbs. per ton constant throughout the period of braking.

The train friction being 15 lbs. per ton, reduces the effective accelerating force from 100 to 85 lbs. per ton, corresponding to a rate of .927 miles per hour per second.

With this data a simple acceleration curve, as in Fig. 1., may be calculated, the train reaching a maximum speed of 28 miles per hour in 30 seconds, when power is shut off and the train allowed to coast with a retardation due to friction of .1635 miles per hour per second for 31 seconds to the braking line, when it is retarded at a uniform rate of 1.635 miles per hour per second and comes to rest in 75 seconds from time of starting.

Owing to the greater efficiency of the run, the train is allowed to coast after reaching its maximum speed rather than to allow it to continue at a uniform speed with just sufficient power supplied to overcome the train friction loss.

It is obvious that the train could be accelerated at a different rate than that corresponding to 70 lbs. per ton and still make the same length of run in the same time, the length of time occupied in coasting depending upon the rate of acceleration, being a maximum with an infinite rate, that is, with the train starting with a certain initial velocity. A minimum rate of acceleration is reached when no time is left for coasting, that is, when brakes are applied as soon as power is shut off.

In Fig. 2 such a set of curves has been prepared showing a train covering a distance of 2000 feet in 75 seconds, as before, but accelerating at various rates from that corresponding to 62.8 lbs. per ton as a minimum up to an infinite rate, or starting with an initial velocity of 25.2 miles per hour.

As the area enclosed by time as abscissæ and speed as ordinates represents the distance covered, this will be a constant quantity for the fixed distance of 2000 ft. assumed, and curves of Fig. 2 are thus constructed with the same enclosed area for each rate of acceleration. The fact is plainly brought out that with a low rate of acceleration a much higher maximum speed is demanded than would be the case if the rate had been increased, and a curve may be plotted by joining the maximum speeds reached for different accelerating rates, as shown.

A friction of 15 lbs. per ton has been chosen as being that of an average train composed of a motor car and three or four trailers and weighing about 120 tons. With heavier and longer trains this rate may be reduced to as low as 7 or 8 lbs. per ton, while for a motor car alone, the rate may be as high as 30 lbs. per ton, due to friction of motors and gearing. The braking effort of 150 lbs. per ton is also chosen as representing what can be done on an average by a train equipped with air brakes and operating at half the slipping coefficient of the wheels.

It is advisable in rapid transit service to keep the maximum speed reached by the trains as low as possible, as this class of work generally calls for a short time-interval between trains

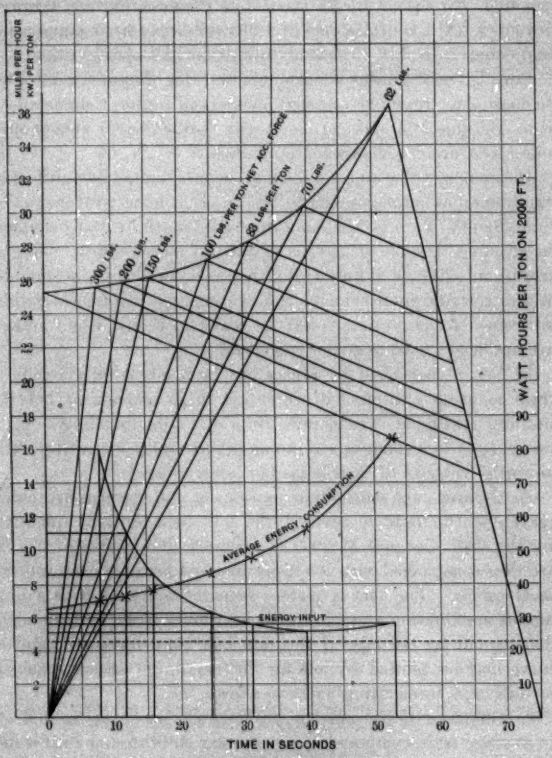

FIG. 2.—Speed and Energy Curves.
Distance, 2,000 feet. Time, 75 seconds. Friction, 15 lbs. per ton. Braking Effort, 150 lbs. per ton.

where the utmost precautions are necessary to keep the trains a safe distance apart. In fact, the maximum speed required for a given run practically determines the time-interval between trains,

as the train headway should be at least five or six times the length of time required to bring a train to rest from its maximum speed with normal braking force applied.

A second reason for higher rates of acceleration lies in the fact that with rates of acceleration approaching the minimum no margin is left for errors of judgment of the motorman, as little or no time is left for coasting, and the rate of acceleration per ampere input is continually varying with the changing passenger load and hence with an overload it becomes difficult to maintain schedule speed.

A third and most important objection to the use of a low rate of acceleration lies in the saving of energy for the run by the use of a higher rate.

Neglecting I^2R and core losses of the motors, a set of energy input curves per ton weight of train may be plotted as in Fig. 2, assuming for simplicity that motors are series wound, operate all in multiple and are so geared that starting resistance is entirely cut out at the various maximum speeds reached in the various runs, that is, that no accelerating is done on the motor curve. A constant impressed E. M. F. is assumed, and starting resistance is supposed to be cut out proportional to the motor speed, thereby keeping the current and torque constant.

Thus it is seen that accelerating a train with the minimum rate calls for the highest maximum speed, demands nearly the least current input and also demands the greatest waste of energy in the brakes, as the speed is a maximum when brakes are applied.

The areas enclosed by the various energy time curves represent the comparative amounts of energy required for the run for the different rates of acceleration and these are plotted in Fig. 2, which compares the energy input and the average energy consumption for the run of 2,000 feet in 75 seconds for all rates of acceleration from the minimum corresponding to 62.8 lbs. per ton up to infinity.

For convenience the average energy rates are plotted in terms of watt hours, and the curve shows that while the minimum rate of acceleration corresponding to 62.8 lbs. per ton calls for an expenditure of 83.5 watt hours per ton weight of train, this is reduced to 56.5 watt hours by accelerating with a rate corresponding to 70 lbs. per ton, to 42.7 watt hours with 100 lbs., and finally reaches a minimum value of 32.5 watt hours if the accelerating rate is pushed to infinity.

Thus the energy required for the run of 2,000 feet in 75 seconds may vary from 83.5 to 32.5 watt hours per ton of train weight, a decrease of over 60 per cent., depending upon the rate of acceleration used.

The curves in Fig. 2 are worthy of careful study, and similar curves afford a means of determining the proper rate of acceleration and hence motor equipment, gearing, etc., to use for a given set of conditions. The limiting factor in the more rapid rate of acceleration of a train is the current input required. Thus, if the rate of acceleration be carried to abnormally high values, the local demand for current becomes so great that either the loss in the feeders more than offsets the reduction in energy consumption at the train, or else the interest on the increased feeder investment is not offset by this energy reduction.

There are other limiting factors governing the rate of acceleration in the size and weight of motors, which are limited in the current they can carry without undue sparking and heating. Thus it will be found that the rate at which a train accelerates, largely determines the cost of feeders, size of motors and generators, both in regard to current and thermal capacity, and also fixes the safe headway between trains.

The error made in accelerating at or near the minimum rate is clearly brought out. For example, the current consumption per ton is the same for 62.8 lbs. per ton or 85 lbs. per ton; as, although 85 lbs. calls for the greater tractive effort, the torque per ampere is so increased by the lesser maximum speed demanded that the current input is the same in each case, hence the feeder considerations are the same in both cases, while the energy consumption shows a reduction from 83.5 watt hours per ton with 62.8 lbs. to 47.5 watt hours, or about half, with 85 lbs. per ton.

Referring again to Fig. 2, it is obvious that a similar set of curves may be plotted for a run of 2,000 feet for any other length of time than 75 seconds, and Fig. 3 gives such a set of curves plotted for lengths of time ranging from 41 seconds, as a minimum possible, up to 210 seconds. The minimum time in which it is possible to make a run of given length is determined by the braking effort, in this case assumed to be 150 lbs. per ton, the train reaching a maximum speed of 66.5 miles per hour in zero seconds with an infinite accelerating force and being retarded throughout the entire running time of 41 seconds at the rate of 1.635 miles per hour per second, corresponding to a force of 150 lbs. per ton braking effort.

The constants assumed in these curves are the same as before, 15 lbs. per ton friction rate, and 150 lbs. per ton braking effort applied uniformly until the train comes to rest at a distance of

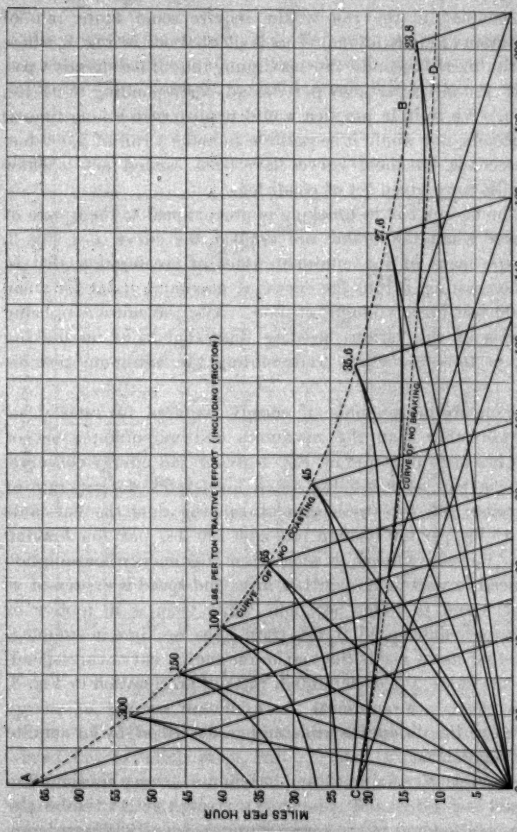

Fig. 3.—Speed Curves. Distance, 2,000 feet. Train Friction, 15 lbs. per ton. Braking Effort, 150 lbs. per ton.

2,000 feet from the start. While the braking effort determines the minimum length of time for the run, the friction rate imposes a limit upon the maximum rate of acceleration possible for

lengths of time greater than 128 seconds for 2,000 foot run. That is, a train accelerating with an infinite rate and coasting the entire length of 2,000 feet would come to rest in 128 seconds with no energy loss in the brakes, and any longer interval of time occupied in the run would require some finite rate of acceleration at a maximum. This is pointed out in Fig. 3, where for a run in 210 seconds the maximum rate of acceleration possible is .175 miles per hour per second, corresponding to 24 lbs. per ton. No train in practice would require such a long time as 210 seconds, nor would it be possible to make a run of 2,000 feet in 41 seconds, but these curves have been carried out to show the limits for a given set of conditions.

As the energy lost in breaking is proportional to the square of the speed when the brakes are applied, the curve A–B, Fig. 3, being the locus of the minimum rates of acceleration, that is, with no coasting, is thus the curve of maximum input for a run of 2,000 feet for any length of time. Also the curve C–D, being the locus of the various coasting lines, that is, of no braking effort, is thus the curve representing the minimum possible input.

To compare the amounts of energy required for rates of acceleration other than the maximum and minimum, a set of curves has been prepared in Fig. 4 giving the energy consumption for a run of 2,000 feet on a level track for any rate of acceleration and for any length of running time, the constants being 15 lbs. per ton friction rate and 150 lbs. per ton braking effort. For convenience in comparison, the energy consumption is reduced to watt hours per ton mile, and speed is expressed as average speed in miles per hour while train is in motion or equaling schedule speed if the train loses no time in stopping. The dotted curve A–B is the maximum energy curve corresponding to the curve A–B of minimum rates of acceleration in Fig. 3, and the curve C–D represents the minimum amount of energy possible for the different speeds and is described by an infinite rate of acceleration.

The curves of maximum and minimum energy consumption approach each other and coincide at a speed of 33.3 miles per hour, corresponding to an energy consumption of 298 watt hours per ton mile, this value being the greatest amount of energy that can be expended on the run with 150 lbs. per ton braking effort.

All energy values are net, that is, they represent the amount of energy required to accelerate the train plus energy lost in overcoming friction, and hence take no account of any losses occuring in actual operation in the motors, rheostats, gearing, etc.

Fig. 4 shows the economy resulting from properly proportioning the accelerating rate to the schedule speed and distance traveled. For example, a train accelerating with a tractive

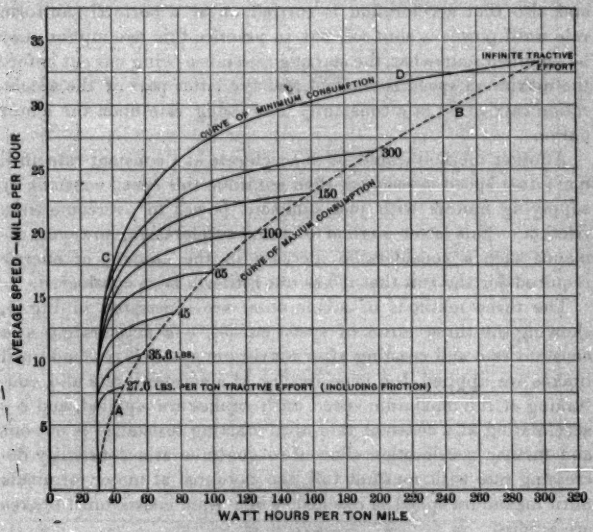

FIG. 4.—Speed-Energy Curves.
Distance, 2,000 feet. Friction, 15 lbs. per ton.

effort of 100 lbs. per ton and making 20 miles per hour average speed, not including stops, will require 127 watt hours per ton mile, which would be reduced to 65 watt hours per ton mile if the tractive effort had been increased to 150 lbs. per ton, or, in other words, the generator capacity would be but half as large for the same service.

The curve of 300 lbs. per ton tractive effort is interesting as it represents about the maximum speed attainable with modern

apparatus for a distance of 2000 ft. with the assumed constants of friction and braking effort. Assuming the entire weight of the car to rest upon drivers, about 300 lbs. per ton would be available for traction without slipping the wheels on an average track, so that an average speed of 26½ miles per hour is the highest that could be obtained over a distance of 2000 ft., not allowing any time whatever for coasting.

All previous curves have been based upon the assumption that trains are allowed to coast after reaching maximum speed, and also that acceleration is carried on at a perfectly uniform rate until power is shut off, but in practice this assumption may be modified somewhat, the starting resistance being cut out before the maximum speed is reached, and the latter part of the acceleration carried on at a constantly decreasing rate upon the motor curve.

Another method would be to accelerate at a constant rate until maximum speed is reached, then continue this speed constant by supplying motors with just sufficient power to overcome train friction. This latter method of train acceleration, however, demands such a considerable increase in the amount of energy required for the run that it has not hitherto been considered.

The three methods of acceleration are illustrated in Fig. 5, showing the three forms of speed curves, *a–a* accelerating at a constant rate and coasting after maximum speed is reached until brakes are applied, *b–b* accelerating at a constant rate and continuing at full maximum speed until brakes are applied, and *c–c* accelerating at a constant rate until starting resistance is cut out and further acceleration allowed to continue at a constantly decreasing rate with constant full line potential at motor terminals until maximum speed is reached, when train coasts until brakes are applied.

Curve *a–a* reaches the highest maximum speed but wastes the least energy in the brakes, and hence is the most efficient run mechanically, curve *b–b*, the constant speed method, being the least efficient.

As these three curves were plotted from the speed torque curves of an actual motor, it is instructive to compare the watt hours consumed for each run with series parallel control, operating two motors in series, then in multiple. To this end a set of energy input curves have been plotted in Fig. 5, showing that curve *c–c* requires the least maximum energy input, while curve *a–a* requires the greatest amount.

The area enclosed by the energy time curves is a measure of the average energy consumption for each run, and their respective values reduced to watt hours are:

a. Constant current and coasting, 147 watt hours per ton,

b. Constant current and no coasting, 160 watt hours per ton.

c. Constant current and acceleration on motor curve, 126 watt hours per ton.

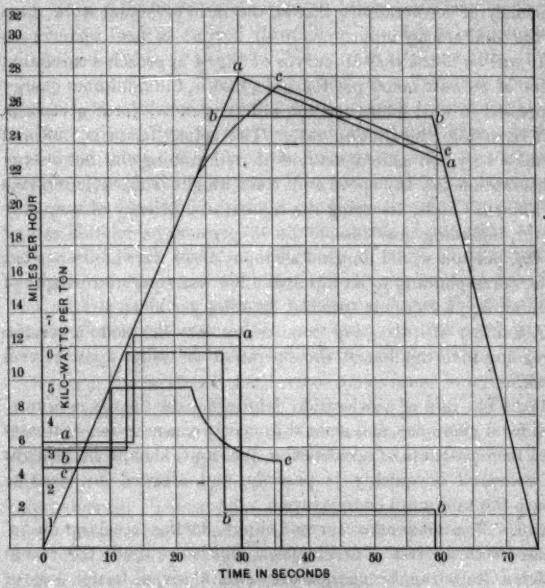

FIG. 5.—Speed Curves.
Distance, 2,000 feet. Time, 75 seconds. Friction, 15 lbs. per ton. Braking Effort, 150 lbs. per ton.

Hence, of the three methods, curve *c*, making use of the acceleration due to a series motor curve, not only requires the least maximum current input, but also requires the least average energy input to the motors for a given run and hence is the form of curve used in the majority of actual runs. Although the energy given out by the motors in run *c* is greater than in run *a*, as evi-

denced by the higher speed at which brakes are applied, yet this extra work is done so much more efficiently, owing to the smaller starting resistance loss, that the total watt hours input becomes less.

Whatever form of acceleration curve be used, the results arrived at in Fig. 4 will not be greatly modified if similar forms of curves are compared, and, being expressed in terms of actual work done, may be used for any run with a factor expressing the efficiency of acceleration: that is, the ratio of actual work done to the total energy input.

It will be noted that all curves of Fig. 4 approach a minimum value of 30 watt hours per ton mile, that is, the minimum energy expressed in watt hours per ton mile expended for a given run will be double the friction rate. The actual factor is 1.98 and forms a very convenient method of determining the net energy consumption for any speed and train weight if the friction rate be known. Thus, assuming the light load efficiency of a railway motor, including gear loss, to be 75 per cent., a friction rate of 15 lbs. per ton would demand an input of 40 watt hours per ton mile, corresponding to an input of 1200 watts per ton weight of train at, say, a constant speed of 30 miles per hour.

A number of interesting conclusions may be made from the foregoing investigation of the operation of trains upon a level track.

1st. The rate of acceleration determines the energy consumption for a given run, and since this energy consumption decreases with increased rate of acceleration, the train should be brought up to speed as quickly as possible and allowed to coast to secure the minimum energy input.

2nd. The maximum current input during acceleration increases with the rate of acceleration, and hence limits the rate at which a train can be accelerated with a given feeder loss or feeder investment.

3rd. In order to reduce the average energy consumption and also the maximum current input to a minimum for a given run, a due amount of acceleration should take place on the motor curve after starting resistance is cut out, hence a motor should be carefully proportioned for the work it has to do.

4th. A normal amount of coasting should be permitted after power is shut off, partly to provide a margin to allow for errors of judgment of the motorman, but largely because this is the

most efficient method of accelerating a train. On no account should the maximum speed be continued by supplying the motors with just sufficient current to overcome train friction, as this method of accelerating is extremely wasteful and inefficient.

Having discussed various methods of accelerating, it is interesting to follow out the problem, and determine the actual efficiency of transporting passengers by our modern methods of travel. It has been pointed out that only from 15 to 20 per cent. of a fully loaded train consists of a paying load, and with an average load as carried throughout the day this percentage will be reduced to 10 per cent. or less, that is, nine-tenths of the energy consumed in moving this train at a constant speed is wasted. But in rapid transit work a train seldom attains a constant speed, due to the frequent stops and high schedule speed, and at least ten times the energy required to overcome friction alone must be expended in accelerating the train, only to appear as heat in the brake shoes when bringing the trains to rest. That is, considering the friction work of the train as the only useful work done, an efficiency of only 10 per cent. is reached in the average run. But only 10 per cent. of this friction work is useful in moving passengers, hence the actual passenger efficiency is reduced to less than one per cent. of the total energy delivered to the train during acceleration.

When it is considered that further losses occur in operation in the motors and their method of control, in the transmission lines and generators of an electric traction system, it will be appreciated that the present method of transportation, with its efficiency of a fraction of one per cent., opens a wide field for improvement. This applies with greater force to rapid transit service using steam locomotives as a motive power, as the dead weight carried per passenger is greater with a steam locomotive than with a motor car, and the efficiency from the coal pile is much less. Hence some means of reducing the large loss, due to accelerating the train is desirable, and this is found in the adoption of an artificial profile, as followed out in a large underground road now building, using down-grades in starting, and up-grades to retard the train when stopping.

The ideal profile would provide for a down-grade at starting sufficient to do the work of acceleration and with an up-grade to do the entire braking required, thus leaving only the friction energy to be supplied by the motive power. Such a road would

operate at 100 per cent. efficiency, neglecting the dead weight of train carried and considering total friction work as useful work.

Reduced to practice, this ideal grade must be modified considerably. The per cent. grade used is limited partly by the rapidity of acceleration that may be imparted to the train without discomfort to the passengers and partly by the available tractive effort of the motive power, which must be sufficient to haul the train up the grade in case of necessity.

A second modification occurs in the necessity of having stations placed on a level track, and since a train has an appreciable length, it must travel its own length before the last car is off the level and the full effect of the grade is felt. Thus, during the first period of acceleration the rate is comparatively small and must be furnished by the motors, resulting in a material reduction in the energy gain in the theoretically ideal profile. An artificial profile can only be secured on elevated or underground roads, and a double track road necessitates two separate overhead structures and the underground road must consist of two separate tunnels with unlike profiles.

Assuming two stations to be on the same level, there are two forms of profile that may be assumed with a given maximum difference in levels, as shown in Fig. 6. Profile A consists of a down-grade with a level track at each terminus equal to the length of train operating over the road, while profile B consists of a down-grade and up-grade equal in length and percentage with a level track connecting them, and also a level track at each station equal to a train length.

In practice a perfectly symmetrical profile, as in B, will not be possible, as the level tract connecting grades must be replaced by a slight grade to provide for drainage in tunnel roads, this grade preferably opposing the direction of movement of the train.

In order to ascertain the behavior of a train upon the two forms of profiles and to determine the most efficient form of grade, a series of curves have been plotted upon the following assumptions:

Length of run total, 2,000 feet.

Train friction, 15 lbs. per ton constant.

Braking effort, 150 lbs. per ton during time brakes are set.

Running time, 75 seconds.

Length of train, 200 feet, corresponding to an average train of four to five cars.

Tractive effort, as supplied by motors, 100 lbs. per ton. (This effort being supplied irrespective of added effort due to grades.)

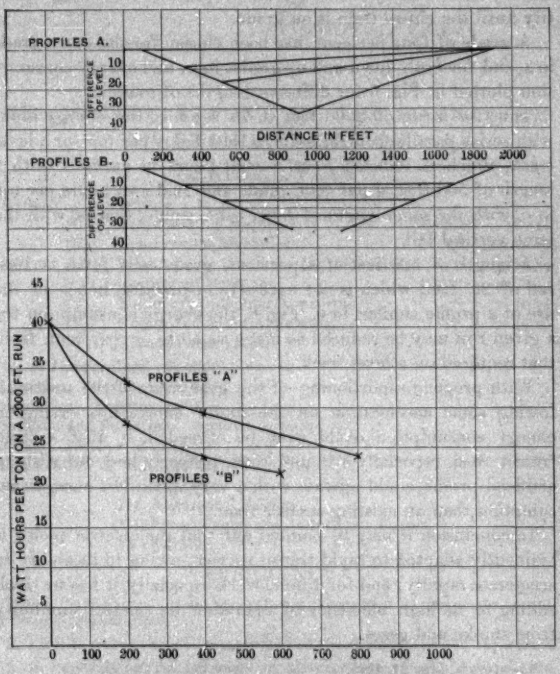

FIG. 6.—Energy Curves on Artificial Profile.
Distance, 2,000 feet. Time, 75 Seconds. Starting Grade of 4%. Tractive Effort of Motors, 100 lbs. per ton. Friction, 15 lbs. per ton. Braking Effort, 150 lbs. per ton.

No running is assumed to take place on the motor curve, and train is supposed to coast after reaching maximum speed. Motors are controlled by the ordinary series-parallel method, starting with two motors in series and throwing them

into multiple, the starting resistance being supposed to be cut out uniformly so that 100 lbs. per ton motor effort is maintained constant while current is on.

Due regard is paid to the fact that the tractive effort due to grade depends upon the proportional length of train off the level track at the stations, and hence is a constantly increasing quantity until the entire train is on grade.

A grade of four per cent. has been chosen for the down-grade in A, and for both down and up grades in B; and energy consumption plotted in Fig. 6 for different lengths of grade.

Thus for a run of 2000 feet in 75 seconds, the energy input with series parallel control is 40.75 watt hours per ton for a level track, which is reduced to 27.8 watt hours in curve A with a length of 500 feet, 4 per cent. grade and 22.6 watt hours per ton in C, with the same length of 4 per cent. gradé, that is, with the same vertical fall.

A length of 500 feet of 4 per cent. grade calls for a vertical fall of 20 feet, which is not excessive in practice, hence by the use of a profile similar to B, Fig. 6, the energy consumption for a given run may be reduced as much as 40 to 50 per cent. from that required on a level track.

With proper proportioning of the gear ratio of the motor allowing some acceleration on the motor curve, this saving in energy consumption could even be exceeded, so that a rapid transit road, especially a tunnel road, properly laid out with an artificial profile, could operate with a very much less energy consumption than an existing surface road.

In conclusion it may be pointed out that the electric motor is eminently adapted to rapid transit service, owing to its ability to accelerate rapidly; and for tunnel work especially, it has no rival, adding to its high efficiency of operation its perfect immunity from smoke and gases.

Schenectady, May 31, 1898.

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Ralph W. Pope, *Secretary.*

NEW YORK, June 21, 1898.

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PRELIMINARY REPORT OF THE COMMITTEE ON STANDARDIZATION.

GENERAL PLAN.

Efficiency. Sections 1 to 30

(I)	Commutating Machines,	Sections 6 to 14
(II)	Synchronous Machines,	" 15 to 16
(III)	Synchronous Commutating Machines,	" 17 to 19
(IV)	Rectifying Machines,	" 20
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Rise of Temperature. Sections 31 to 35
Insulation. Sections 36 to 41
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Variation and Pulsation. Sections 60 to 63
Rating. Sections 64 to 69
Classification of Voltages and Frequencies.
Appendices. (I) Efficiency.
(II) Apparent Efficiency.
(III) Power Factor and Inductance Factor.
(IV) Notation.

"EFFICIENCY."

1. The "efficiency" of an apparatus is the ratio of its net power output to its gross power input.[1]

2. Electric power should be measured at the terminals of the apparatus.

3. In determining the efficiency of alternating-current apparatus, the electric power should be measured when the current is in phase with the E. M. F., unless otherwise specified, except when a definite phase difference is inherent in the apparatus, as in induction motors, etc.

4. Mechanical power in machines should be measured at the pulley, gearing, coupling, etc., thus excluding the loss of power in said pulley, gearing or coupling, but including the bearing friction and windage. The magnitude of bearing friction and windage may be considered as independent of the load. The loss of power in the belt and the increase of bearing friction due to belt tension, should be excluded. Where, however, a machine is mounted upon the shaft of a prime mover, in such a manner that it cannot

1. An exception should be noted in the case of storage batteries or apparatus for storing energy.

be separated therefrom, the frictional losses in bearings which ought, by definition, to be included in determining the efficiency, should be excluded, owing to the practical impossibility of determining them satisfactorily. The windage, however, should be included.

5. The efficiency may be determined by measuring all the losses individually and adding their sum to the output to derive the input, or subtracting their sum from the input to derive the output. All losses should be measured at, or reduced to, the temperature assumed in continuous operation, or in operation under conditions specified.

In order to consider the application of the foregoing rules to various machines in general use, the latter may be conveniently divided into classes as follows:

I. Commutating Machines.

6. Under this head may be classed the following: Direct-current generators; direct-current motors; direct-current boosters; motor-generators; dynamotors; converters and closed-coil arc machines.

7. A booster is a machine inserted in series in a circuit to change its voltage.

8. A motor-generator is a transforming device consisting of two machines; a motor and a generator, mechanically connected together.

9. A dynamotor is a transforming device combining both motor and generator action in one magnetic field, with an armature having two separate windings.

10. A converter is a transforming device combining both motor and generator action in one magnetic field, and in an armature winding common to both.

11. In commutating machines the losses are:—

a. Bearing friction and windage. (See Section 4.)

b. Molecular magnetic friction, and eddy currents in iron and copper. These losses should be determined with the machine on open circuit, and at a voltage equal to the rated voltage $+Ir$ in a generator, and $-Ir$ in a motor, where I denotes the current strength, and r denotes the internal resistance of the machine. They should be measured at the correct speed and voltage, since they do not usually vary in proportion to the speed or to any definite power of the voltage.

c. Armature resistance losses, I^2r', where I is the current strength in the armature, and r' is the resistance between armature brushes, excluding the resistance of brushes and brush contacts.

d. Comutator brush friction.

e. Commutator brush-contact resistance. It is desirable to point out that with carbon brushes the losses (*d*) and (*e*) are usually considerable in low-voltage machines.

f. Field excitation. With separately excited fields, the loss of power in the resistance of the field coils alone should be considered. With shunt fields or series fields, however, the loss of power in the accompanying rheostat should also be included.

(*b*) and (*c*) are losses in the armature or "armature losses;" (*d*) and (*e*) "commutator losses;" (*f*) "field losses."

12. The difference between the total losses under load and the sum of the losses above specified should be considered as "load losses," and, being usually trivial, should be neglected in commutating machines.

13. Boosters should be considered and treated like other direct-current generators in regard to losses.

14. In motor-generators, dynamotors or converters, the efficiency is the $\frac{\text{electric output}}{\text{electric input}}$. In direct-current converters, the I^2r loss of the armature is that due to the difference between the current input and current output.

II. Synchronous Machines.—

15. These include:—Alternators, synchronous alternating-current motors, synchronous compensators and exciters of induction generators.

Synchronous compensators and exciters of induction generators are synchronous machines producing phase displacement but not power, and will be discussed in Appendix I.

In synchronous machines the output or input should be measured with the current in phase with the terminal E. M. F., *i. e.*, in the non-inductive condition, except when otherwise expressly specified.

16. The losses in synchronous machines are :

a. Bearing friction and windage ; see Sec. 4.

b. Molecular magnetic friction and eddy currents in iron, copper and other metallic parts. These losses should be determined at open circuit of the machine at the rated speed and at the rated voltage. $+ I r$ in a synchronous generator, $- I r$ in a synchronous motor, where I = current in armature, r = armature resistance. It is undesirable to compute these losses from observations made at other speeds or voltages.

These losses may be determined either by driving the machine by a motor, or by running it as a synchronous motor, and adjusting its fields so as to get minimum current input and measuring the input by wattmeter. The former is the preferable method, and in polyphase machines the latter method is specially liable to give erroneous results in consequence of unequal distribution of currents in the different circuits caused by inequalities of the impedance of connecting leads, etc.

c. Armature-resistance loss, which may be expressed by $p I^2 r$; where r = resistance of one armature circuit, I = the current in such armature circuit, and p = the number of armature circuits.

d. Load losses as defined in section 12. While these losses cannot well be determined individually, they may be considerable and, therefore, their joint influence should be determined by observation. This can be done by operating the machine on short circuit and at full-load current, that is, by determining what may be called the "short-circuit core loss." With the low field intensity and great lag of current existing in this case, the load losses are usually greatly intensified.

Thus, if the short-circuit core loss at normal frequency and rated current; that is, the input minus friction minus $p I^2 r$, does not exceed 20 per cent of the open-circuit core loss, the load loss can be considered as negligible. Should it be in excess of 20 per cent, then 50 per cent of this excess should be included as load loss.

e. Collector-ring friction and contact resistance. These are generally negligible, except in machines of extremely low voltage.

f. Field excitation. In separately-excited machines, the $I^2 r$ of the field coils proper should be used. In self-exciting machines, however, the loss in the field rheostat should be included.

III. Synchronous Commutating Machines.—

17. These include : 1. Synchronous converters ; *i. e.*, converters from alternating to direct, or from direct to alternating current, and 2. Double-current generators ; *i. e.*, generators producing both direct and alternating currents.

18. The power on the alternating-current side is to be measured with the current in phase with the terminal E. M. F.; *i. e.*, at non-inductive condition.

In double-current generators the efficiency of the machine should be determined as a direct-current generator in accordance with section 11., and as an alternating-current generator in accordance with section 16. The two values of efficiency may be different, and should be clearly distinguished.

19. In synchronous converters the losses are :—

a. Bearing friction and windage, see section 4.

b. Molecular magnetic friction and eddy currents in iron, copper and metallic parts. These losses should be determined at open circuit and at the rated terminal voltage, no allowance being made for the armature resistance, since the alternating and the direct currents flow in opposite directions.

c. Armature resistance. The loss in the armature is $q\ I^2\ r$, where $I =$ direct current in armature, $r =$ armature resistance and q, a factor which is equal to 1.37 in single-phasers, 56 in three-phasers 37 in quarter-phasers and 26 in six-phasers.

d Load losses. Whether the load losses are negligible or not, should be determined in the same manner as described in section 16 *d*.

e and *f*. Losses in commutator and collector friction and brush-contact resistance. See sections 11 and 16.

g. Field excitation. In separately-excited fields, the $I^2\ r$ loss in the field coils proper should be taken, while in shunt and series fields the rheostat loss should be included, except where fields and rheostats are intentionally modified to produce effects outside of the conversion of electric power, as for producing phase displacement for voltage control. In this case 25 per cent. of the $I^2\ r$ loss in the field proper at non-inductive alternating circuit should be added as proper allowance for normal rheostat losses.

Where two equal synchronous converters are available, the efficiency can be determined by operating one machine as a converter from alternating to direct current, the other as a converter from direct to alternating, and supplying the constant losses by an alternator and the variable losses by two low voltage machines inserted between the two machines on their direct-current sides.

IV. Rectifying Machines or Pulsating-Current Generators.—

20. These include : Open-coil arc machines, constant-current rectifiers, constant potential rectifiers.

The losses in open-coil arc machines are essentially the same as in sections 6 to 14 (closed-coil commutating machines.) In alternating-current rectifiers, however, the output must be measured by wattmeter and not by voltmeter and ammeter, since owing to the pulsation of current and E. M. F., a considerable discrepancy may exist between watts and volt-amperes, amounting to as much as 10 or 15 per cent.

In constant-current rectifiers, transforming from constant-potential alternating to constant direct current by means of constant-current transformers and rectifying commutators, the losses in the transformers are to be included in the efficiency and have to be measured when operating the rectifier, since in this case the losses are generally greater than when feeding an alternating secondary circuit. In constant-current transformers the load losses are usually larger than in constant-potential transformers and thus should not be neglected.

The most satisfactory method of deriving the efficiency in rectifiers is to measure electric input and electric output by wattmeter The input is usually not non-inductive owing to a considerable phase displacement and to wave distortion. The power consumed by the synchronous motor or other source driving the rectifier should be included in the electric input.

V. Stationary Induction Apparatus.—

21. These include : Transformers, compensators or auto-transformers, potential regulators of the induction or magneto type, and reaction coils.

Regarding reaction coils as apparatus used to produce phase displacement see appendix I.

22. Compensators or auto-transformers are transformers using parts of the same coil both as primary and as secondary. Potential regulators are transformers or compensators inserted in series in a system to vary its voltage.

23. Since the efficiency of induction apparatus depends upon the wave shape of E. M. F., it should be referred to a sine wave of E. M. F., except where expressly specified otherwise. The efficiency should be measured with non-inductive load, and at rated frequency, except where expressly specified otherwise. The losses are:

a. Molecular magnetic friction and eddy currents measured at open circuit and at rated voltage — Ir, where I = rated current, r = resistance of primary circuit.

b. Resistance losses, the sum of the $I^2 r$ of primary and of secondary in a transformer or of the two sections of the coil in the compensator or auto-transformer, where I = current in the coil or section of coil, r = resistance.

c. Load losses; *i. e.* eddy currents in the iron and especially in the copper conductors, caused by the current. They should be measured by short-circuiting the secondary of the transformer and impressing upon the primary an E. M. F. sufficient to send full-load current through the transformer. The loss in the transformer under these conditions measured by wattmeter gives the load losses + $I^2 r$.

d. Losses due to the methods of cooling, as power consumed by the blower in air-blast transformers, and power consumed by the motor driving pumps in oil or water-cooled transformers. Where the same cooling apparatus supplies a number of transformers or is installed to supply future additions, allowance should be made therefor.

24. In potential regulators the efficiency should be taken at the maximum voltage for which the apparatus is designed, and with non-inductive load.

VI. Rotary Induction Apparatus.

25. These comprise:—Induction motors, induction generators, frequency changers, and rotary-phase transformers.

26. Owing to the existence of load losses and since the magnetic density in the induction motor under load changes in a complex manner, the efficiency should be determined by measuring the electric input by wattmeter and the mechanical output at the pulley, gear, coupling, etc.

27. The efficiency should be determined at the rated frequency and the input measured with sine waves of impressed E. M. F.

28. The efficiency may be calculated from the current input and power input,

a. Running light, and,

b. Standing still, by calculating the induction motor load curves, provided that over the whole range of the load curves, the slip of speed and the current coincide with the values found by said tests and that the calculated maximum output coincides with tests also.

The same applies to induction generators.

29 In frequency changers; *i. e.*, apparatus transforming from a polyphase system to an alternating system of different frequency, with or without a change in the number of phases, and rotary-phase transformers; *i. e.*, apparatus transforming from an alternating system, usually single phase, to another alternating system usually polyphase, of the same frequency, the efficiency should also be determined by measuring both input and output.

VII. Transmission Lines.

30. The efficiency of transmission lines should be measured with non-inductive load at the receiving end, with the rated receiving pressure and frequency, also with sinusoidal impressed E. M. F.'s., except where expressly specified otherwise, and with the exclusion of transformers or other apparatus at the ends of the line.

RISE OF TEMPERATURE.

General Principles.—

31. Under regular service conditions, the temperature of electrical machinery should never be allowed to remain at a point at which permanent deterioration of its insulating material takes place.

32. The rise of temperature should be referred to the standard conditions of a room-temperature of 25° *C.*, a barometric pressure of 760 mm. and normal conditions of ventilation; that is, the apparatus under test should neither be exposed to draught, nor enclosed, except where expressly specified.

If the room temperature during the test differs from 25° *C.*, the observed rise of temperature should be corrected by 1/2 per cent for each degree *C.* Thus, with a room temperature of 35° *C.*, the observed rise of temperature has to be decreased by 5 per cent, and with a room temperature of 15° *C.*, the observed rise of temperature has to be increased by 5 per cent. The thermometer indicating the room temperature should be screened from thermal radiation emitted by heated bodies.

33. The temperature should be measured after a run of sufficient duration to reach practical constancy. This is usually from 6 to 18 hours, according to the size and construction of the apparatus. It is permissible, however, to shorten the time of the test by running a lesser time on an overload, then reducing the load to normal, and maintaining it thus until the temperature has become constant.

In apparatus intended for intermittent service, as railway motors, starting rheostats, etc., the rise of temperature should be measured after a shorter time, depending upon the nature of the service, and should be specified.

In apparatus which by the nature of their service may be exposed to overload, as railway converters, and in very high voltage circuits, a smaller rise of temperature should be specified than in apparatus not liable to overloads or in low-voltage apparatus. In apparatus built for conditions of limited space, as railway motors, a higher rise of temperature must be allowed.

34. In electrical conductors, the rise of temperature should be determined by their increase of resistance. For this purpose the resistance may be measured either by galvanometer test, or by drop-of-potential method. A temperature coefficient of 0.4 per cent per degree C., may be assumed for copper. Temperature elevations measured in this way are usually in excess of temperature elevations measured by thermometers.

35. The following maximum values of temperature elevation are recommended:

Commutating machines, rectifying machines, and synchronous machines.
Field and armature, by resistance, 50° C.
Commutator and collector rings, by thermometer, 55° C
Bearings and other parts of machine, by thermometer, 40° C.
Rotary induction apparatus, electric circuits, 50° C., by resistance.
Bearings and other parts of the machine 40° C., by thermometer.

In squirrel cage or short-circuited armatures, 60° C., by thermometer, may be allowed.

Transformers of over 15 K. W. capacity — electric circuits by resistance 50° C., other parts by thermometer, 40° C., under conditions of normal ventilation.

Reactive coils, induction and magneto regulators and transformers of 15 K. W. or less — electric circuits by resistance 55°C., other parts by thermometer 45°C.

In apparatus intended for intermittent service from 10 to 20° more rise under continuous operation can be allowed.

Railway motors — 75°C. after one hour's operation at full load.

INSULATION.

36. The ohmic resistance of the insulation is of secondary importance only, as compared with the dielectric strength or resistance to rupture by high voltage.

Since the ohmic resistance of the insulation can be very greatly increased by baking, but the dielectric strength is liable to be weakened thereby, it is preferable to specify a high dielectric strength rather than a high insulation resistance.

Insulation Resistance.

37. Insulation resistance tests should, if possible, be made at the pressure for which the apparatus is designed.

The insulation resistance must be such that the rated voltage of the apparatus will not send more than $\frac{1}{100{,}000}$ of the full load current, at the rated terminal voltage, through the insulation. Where the value found in this way exceeds 1 megohm, 1 megohm should be considered as sufficient.

Dielectric Strength.

38. The dielectric strength or resistance to rupture should be determined by a continued application of an alternating E. M. F. for 5 minutes. The source of alternating E. M. F. should be a transformer of such size that the charging current of the apparatus as a condenser does not exceed 25% of the rated capacity of the transformer.

39. The test for dielectric strength should be made with the completely assembled apparatus and not with its individual parts, and the voltage should be applied as follows:—

1st. Between electric circuits and surrounding conducting material, and,

2nd. Between adjacent electric circuits, where such exist, as in transformers.

The tests should be made with a sine wave of E. M. F., or where this is not available, at a voltage giving the same striking distance between needle points in air, as a sine wave of the specified E. M. F., except where expressly specified otherwise. As needles, new sewing needles should be used. It is recommended to shunt the apparatus during the test by a spark gap of needle points set for a voltage exceeding the required voltage by 10%.

40. The following voltages are recommended:

In electric circuits of rated voltage up to 500 volts.

	VOLTS.
In apparatus of 10 K. W. capacity and less	1000
In apparatus over 10 K. W. capacity	1500
Of rated voltages over 500, but less than 1000 volts:	
In apparatus of 10 K. W. capacity and less	2000
In apparatus over 10 K W. capacity	3000
1000 and more, but less than 2500 volts	5000
2500 " " " " " 3500 volts	7000
3500 " " " " " 6600 volts	10000
6600 " "	$1\frac{1}{2}$ × rated voltage.
Alternator fields in machines of over 1000 volts	3000
Synchronous motor and rotary converter fields	5000

Condensers should be tested at twice their rated voltage and at their rated frequency.

The above values are effective values, or square roots of mean square reduced to a sine wave of E. M. F.

Synchronous motors and synchronous converter field coils should be tested at 5000 volts, since in the starting of such machines a high voltage is induced in their field coils.

41. In testing insulation between different electric circuits as between primary and secondary of transformers, the testing voltage must be chosen corresponding to the high-voltage circuit.

In transformers of 10,000 volts or more, it should be considered as sufficient to operate the transformer at 1.5 × its rated voltage, either by increasing the magnetic density, or by increasing the frequency and connecting first the one, and then the other terminal to the core, with respect to the secondary winding.

REGULATION.

42. The term regulation should have the same meaning as the term "inherent regulation," at present frequently used.

43. The regulation of an apparatus intended for the generation of constant potential, constant current, constant speed, etc., is to be measured by the maximum variation of potential, current, speed, etc, occurring within the range from full load to no load, under such constant conditions of operation as give the required full-load values.

44. The regulation is given in percentage of the full-load value of potential, current, speed, etc., and the apparatus should be steadily operated during the test under the same conditions as at full load.

45. The regulation of generators is to be determined at constant speed; of alternating apparatus at constant impressed frequency.

46. In apparatus generating, transforming or transmitting alternating currents, regulation should be understood to refer to non inductive load, that is to a load in which the current is in phase with the E. M. F. at the output side of the apparatus, except where expressly specified otherwise.

47. In alternating apparatus receiving electric power, regulation should refer to a sine wave of E. M. F., except where expressly specified otherwise.

48. In commutating machines, rectifying machines and synchronous machines, as direct-current generators and motors, alternating-current and polyphase generators, the regulation is to be determined under the following conditions:

a. At constant excitation in separately excited fields

b. With constant resistance in shunt field circuits and

c. With constant resistance shunting series fields; *i. e.*, the field adjustment should remain constant, and should be so chosen as to give the required full-load voltage at full-load current.

49. In constant-potential machines, the regulation is the ratio of the maximum difference of terminal voltage from the rated full-load value (occurring within the range from full load to open circuit) to the full-load terminal voltage.

50. In constant-current machines, the regulation is the ratio of the maximum difference of current from the rated full-load value (occurring within the range from full load to short circuit), to the full-load current.

51. In constant-power machines, the regulation is the ratio of maximum difference of power from the rated full load value occurring within the range of operation specified) to the rated power.

52. In over-compounded machines, the regulation is the ratio of the maximum difference in voltage from a straight line connecting the no-load and full-load values of terminal voltage as function of the current, to the full-load terminal voltage.

53. In constant-speed continuous current motors the regulation is the ratio of the maximum variation of speed from its full load value (occurring within the range from full-load to no-load) to the full-load speed.

54 In transformers, the regulation is the ratio of the rise of secondary terminal voltage from full-load to no-load, (at constant primary impressed terminal voltage) to the secondary terminal voltage.

55. In induction motors, the regulation is the ratio of the rise of speed from full-load to no-load, (at constant impressed voltage), to the full-load speed

56. In converters, dynamotors, motor-generators, and frequency changers, the regulation is the ratio of the maximum difference of terminal voltage at the output side from the rated full-load voltage, (at constant impressed voltage and at constant frequency), to the full-load voltage on the output side.

57. In transmission lines, feeders, etc., the regulation is the ratio of maximum voltage difference at the receiving end, between no-load and full non-inductive load, to the full-load voltage at the receiving end, with constant voltage impressed upon the sending end.

58. In steam engines, the regulation is the ratio of the rise of speed from full-load to no load (at constant steam pressure at the throttle), to the full-load speed.

59. In a turbine or other water-motor, the regulation is the ratio of the rise of speed from full-load to no-load (at constant head of water; *i. e.*, at constant difference of level between tail race and head race), to the full-load speed.

Variation and Pulsation.—

60. In prime movers which do not give an absolutely uniform rate of rotation or speed, as in steam engines, the "variation" is the maximum displacement in position of the revolving member from the position it would occupy at uniform rotation, expressed in degrees circumference, that is with one revolution as 360°, and the pulsation is the ratio of the maximum change of speed in an engine cycle to the average speed.

61. In alternators or alternating-current circuits in general, the variation is the maximum difference in phase of the generated wave of E. M. F. from a wave of absolutely constant frequency, expressed in degrees, and is due to the variation of the prime mover. The pulsation is the ratio of the maximum change of frequency during an engine cycle to the average frequency.

62. If n = number of poles, the variation of an alternator is $\frac{n}{2}$ times the variation of its prime mover if direct connected, and $\frac{n}{2}\phi$ times the variation of the prime mover if connected thereto by belting, gearing, etc., in the velocity ratio ϕ.

63. The pulsation of an alternating-current circuit is the same as the pulsation of the prime mover of its alternator.

64. Both electrical and mechanical power should be expressed in kilowatts, except when otherwise specified. Alternating-current apparatus should be rated in kilowatts on the basis of non-inductive condition; *i. e.*, with the current in phase with the terminal voltage.

65. Thus the electric power generated by an alternating-current apparatus equals its rating only at non-inductive load, that is when the current is in phase with the terminal voltage.

66. The full-load current of an electric generator is that current which with the rated full-load terminal voltage gives the rated kilowatts, but in alternating-current apparatus only at non-inductive load.

67. Thus in machines in which the full load voltage differs from the no-load voltage, the full-load current should refer to the former.

If P = rating of an electric generator and E = full-load terminal voltage, the full-load current is:

$I = \frac{P}{E}$ in a continuous-current machine or single phase-alternator.

$I = \frac{P}{E\sqrt{3}}$ in a three-phase alternator.

$I = \frac{P}{2E}$ in a quarter-phase alternator.

68. Constant-current machines, such as series arc-light generators, should be rated in kilowatts based on terminal volts and amperes at full load.

69. The rating of a fuse or circuit breaker should be the current-strength at which it will open the circuit, and not the working-current strength.

RATING.

Classification of Voltages and Frequencies.

70. In continuous-current, low-tension generators, the following terminal voltages are in general use and are recommended:

125 volts. 250 volts. 550 volts.

71. In continuous-current, low-tension motors, and other receiving apparatus the following terminal voltages are in general use and are recommended:

110 volts, 220 volts. 500 volts.

72. In alternating-current, low-tension consumption circuits the following pressures are in general use, and are recommended:

110 volts. 220 volts. 440 volts.

73. In alternating-current, high-tension circuits at the receiving end, the following pressures are in general use, and are recommended:

1000 volts. 2000 volts. 3000 volts. 6000 volts
10000 volts. 15000 volts. 20000 volts.

74. In alternating-current high-tension generators, the following terminal voltages are in general use and are recommended:

1150 volts 2300 volts 3450 volts 6600 volts
11000 volts 16500 volts 22000 volts.

75. In alternating-current circuits, the following frequencies are recommended:

25 ~ 40 ~ 60 ~ 120 ~

APPENDIX I.

EFFICIENCY.

Efficiency of Phase-Displacing Apparatus.

In apparatus producing phase displacement as, for example, synchronous compensators, exciters of induction generators, reactive coils, condensers, polarization cells, etc., the efficiency should be understood to be the ratio of the volt-ampere activity minus power loss, to the volt-ampere activity.

The efficiency may be calculated by determining the losses individually, substracting them from the volt-ampere activity, and then dividing by the volt-ampere activity.

1st. In synchronous compensators and exciters of induction generators, the determination of losses is the same as in other synchronous machines under Sections 15 and 16.

2nd. In reactive coils the losses are molecular friction, eddy losses, and I^2r loss. They should be measured by wattmeter. The efficiency of reactive coils should be determined with a sine wave of impressed E. M. F., except where expressly specified otherwise.

3rd. In condensers, the losses are due to dielectric hysteresis and leakage and should be determined by wattmeter with a sine wave of E. M. F.

4th. In polarization cells, the losses are those due to electric conductivity and a loss in the electrolyte of the nature of chemical hysteresis and are usually very considerable. They depend upon the frequency, voltage and temperature.

APPENDIX II.

Apparent Efficiency.

In apparatus in which a phase displacement is inherent to their operation, apparent efficiency should be understood as the ratio of net power output to volt-ampere input.

Such apparatus comprise induction motors, reactive synchronous converters, synchronous converters controlling the voltage of an alternating current system, self-exciting synchronous motors, potential regulators, and open magnetic circuit transformers, etc.

APPENDIX III.

Power Factor and Inductance Factor.

The power factor in alternating circuits or apparatus may be defined as the ratio of the electric power, in watts, to volt-amperes.

The inductance factor is to be considered as the ratio of wattless volt-amperes to total volt-amperes.

Thus, if p = power factor, q = inductance factor,

then $$p^2 + q^2 = 1$$

The power factor is the $$\frac{\text{(energy component of current or E. M. F.)}}{\text{(total current or E. M. F.)}}$$

and the inductance factor is the

$$\frac{\text{(wattless component of current or E. M. F.)}}{\text{(total current or E. M. F.)}}$$

APPENDIX IV.

The following notation is recommended :—

E, e, voltage, E. M. F., potential difference
I, i, current
P, power
Φ, magnetic flux
B, magnetic density
R, r, resistance
x, reactance
z, impedance
L, inductance
C, capacity.

Vector quantities when used should be denoted by capital italics.

FRANCIS B. CROCKER, *Chairman.*
CARY T. HUTCHINSON.
A. E. KENNELLY.
J. W. LIEB, JR.
CHARLES P. STEINMETZ.
LEWIS B. STILLWELL.
ELIHU THOMSON.

NOTICE.—Communications in discussion of the above report are invited from those not attending the Omaha meeting.

RALPH W. POPE, *Secretary.*

A paper to be presented at the 15th General Meeting of the American Institute of Electrical Engineers, Omaha, June 28th, 1898.

[ADVANCE COPY—SUBJECT TO REVISION.]

POWER TRANSMISSION AND DISTRIBUTION FOR RAILWAY WORK.

BY ERNST J. BERG.

When electric power was first applied to railway work, the distances of transmission were comparatively short, so that one generating station usually located in the center of distribution was sufficient to furnish all power at a reasonable loss.

As the plants were extended it was soon evident that even disregarding the question of efficiency, the distances were very limited for commercial and practical reasons, and boosters connected in series with the line so as to raise the voltage proportional to the load were introduced. By their use it was possible to cover considerable distance and yet keep the potential up without too excessive an amount of copper. Since, however, any booster system must be very inefficient, such installations are not very common, and their field of usefulness limited to cases where considerable power is taken and at the end of a long line for a short time only. The next step towards extension was the introduction of power stations located in various places in the network, but the complication and inefficiency of such an arrangement was apparent, and the possibility of transmitting the power by alternating currents was brought out.

The first suggestion was to generate alternating current power in one station and transmit it at a reasonably high voltage to various sub-stations placed where the steam stations would have been located under former conditions, and there convert it to direct current by means of synchronous motors driving direct current generators. This method of transmitting and converting

electrical power was applied in a few cases but was soon superseded by the converters which not only gave a simpler means of converting alternating current to direct current, but which are more

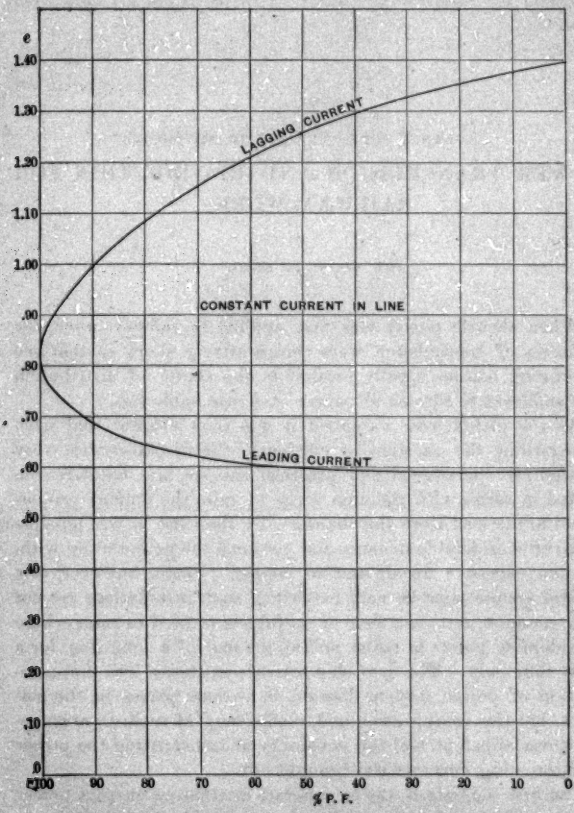


FIG. 1.—Curve Showing how the Generator Voltage Depends upon the Power Factor of the Receiving Circuit. Constant Line Current. Voltage at Receiving End of Transmission Assumed as 1. $Z = .1 - .4j$.

efficient and permit of perfect automatic potential control. Thus in 1894 large and representative converter systems were installed,

Since, with the introduction of long distance transmission the generators were placed far away from the converters and connected to them by lines of considerable resistance and self-induction, it was apparent that potential control could hardly be effected in the power station, at least a control of sufficient sensitiveness to follow the greatly fluctuating loads in a railway circuit, the more so as the voltage does not vary in the same proportion as the load.

To illustrate the variation in voltage at the receiving end of a line of considerable resistance and reactance as function of the phase relation in the current, in Fig. 1 is plotted a curve giving as abscissæ the power factor of the load, that is cos ω and as ordinates the voltage at the receiving end. The line current is the same in all cases, the generator voltage is constant and assumed as unity, the resistance is 10 per cent. and reactance 40 per cent. The upper part of the curve gives the voltage with leading current, the lower with lagging current.

It is readily seen from this diagram how impossible it would be to control the potential at the generating station under these conditions, as, for instance, with a load of 90 per cent. power factor the voltage at the receiving end would be 69 per cent. of the generator voltage with lagging current, and the same as at the generator with leading current. Thus, if the generator should be controlled by the amount of current taken, it is evident that no indications whatever would be made by the current itself, since it is the same under these two conditions.

But even if a wattmeter were in the circuit at the power station, this would not help matters, since under the conditions referred to the watts, that is the power, is the same, and yet the voltage so widely different.

In Fig. 2 this is demonstrated in a slightly different way. It is assumed that the voltage impressed upon the converter shall be constant at all loads, and shows how the generator voltage has to be varied in order to keep this constant at rated output with different power factors, and consequently at a variable line current. It is equally evident from this curve how unfeasible voltage control from the generating station would be.

It was therefore necessary to develop some means by which the potential could be controlled, not at the generator station, as is done in direct current railway circuits, but in the con-

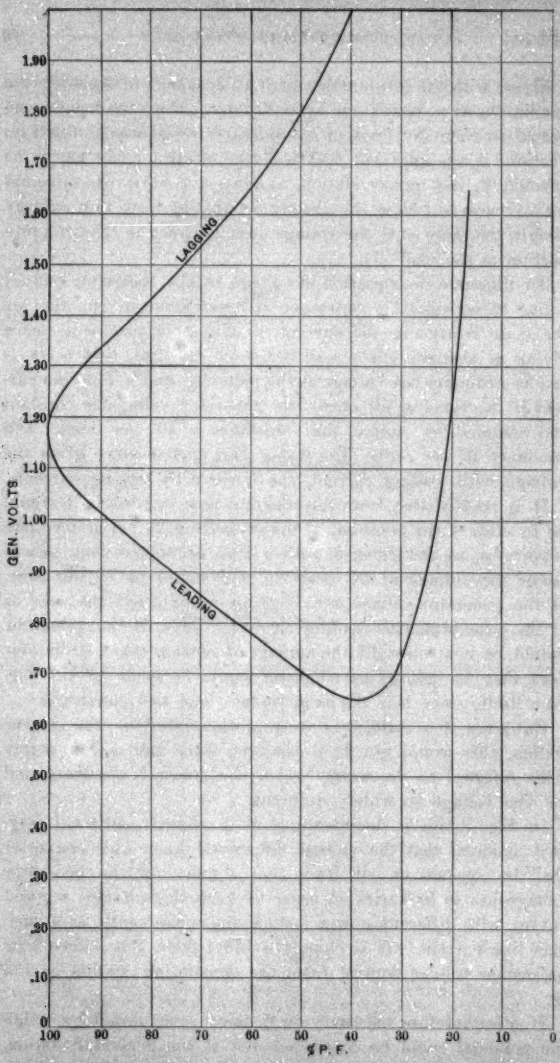


FIG. 2.—Curve Showing how the Generator Voltage Depends upon Power Factor of Load. Constant Load of Receiving Circuit. Voltage at Receiving End of Transmission Assumed as $1.$, $Z = .1 - .4j$.

verter station. This means of regulating the potential was found in changing the phase relation of the current taken by the converter by means of a change of field excitation, raising or lowering the impressed alternating voltage, and consequently the direct current voltage by means of this current passing through lines of considerable self-induction, or through special reactive coils inserted in the circuit.

In the following I shall endeavor to explain this method of control, and discuss its effects on the operation in general.

In a synchronous converter system we have to distinguish between three E. M. F's.; the impressed E. M. F., that is, the E. M. F. at the collector rings of the rotary converter; the counter E. M. F., which is the E. M. F. induced in the rotary converter by the armature revolving in the magnetic field, and consequently proportional to the field excitation; and the E. M. F. of impedance or the E. M. F. consumed by impedance, which is that caused by the current flowing through the reactance and resistance of the system.

The first mentioned E. M. F., that is the impressed E. M. F., is entirely dependent upon the generator voltage; the counter E. M. F. is entirely dependent upon the field excitation of the converter, and is constant regardless of the load on a machine, and the E. M. F. consumed by the impedance changes with the load and is proportional to the current.

To explain the phase relations between these E. M. F's. let us for the sake of simplicity consider the converter running light or doing no work. Since the work done by a synchronous converter rotary is the product of the current taken by the rotary and the projection of the counter E. M. F. on this current at no load the current must be in quadrature, that is, at right angles to the counter E. M. F. Since, furthermore, the input of a rotary converter is expressed by the product of the impressed E. M. F. and the projection of the current on the impressed E. M. F. under the assumption that no energy is consumed, the current is also in quadrature or at right angles to the impressed E. M. F. Consequently the impressed E. M. F. and the counter E. M. F. must be in phase and in opposition to each other. Let, then, the field excitation of the converter be reduced, so that the counter E. M. F. is less than the impressed. Since only three E. M. F's. are acting in the system and their sum must always be zero, that is, the impressed E. M. F. must at any time be equal to the counter E.

E. M. F., plus the E. M. F. consumed by impedance or neglecting the resistance, the E. M. F. consumed by the reactance. The counter E. M. F. is assumed to be less than the impressed, therefore, the E. M. F. consumed by reactance must be in phase with the counter E. M. F. and add itself thereto. But since the current is always 90° ahead of the E. M. F. of self-induction it must be behind 90° the impressed E. M. F. and consequently be lagging.

If, on the contrary, the field excitation of the converter is increased so that its counter E. M. F. is higher than the impressed, it is evident that the E. M. F. consumed by reactance must be in phase with the impressed E. M. F. and add itself thereto. Consequently since the current is 90° ahead of the E. M. F. of self-induction it must be also ahead of the impressed E. M. F., in other words, the current is leading. By a similar discussion it is readily seen that the same argument is true, if the converter is loaded. That is, in this case the wattless lagging or leading current due to the difference between counter E. M. F. and impressed E. M. F., merely adds itself in its proper phase relation to the energy current representing the load.

It is thus seen how by means of converter, leading or lagging currents are caused to flow in the lines. It is readily shown how the voltage at the receiving end of the system can be changed by causing a leading or lagging current to flow over a line with some self-induction.

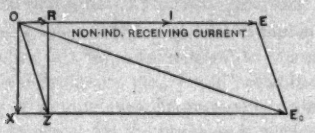
Graphically this is done in Fig. 3. Let OI denote the current which is assumed to be in phase with the E. M. F. in diagram A. The E. M. F. consumed by resistance OR is in phase with this current and the E. M. F. consumed by reactance OX is 90° ahead of the current. Thus, the E. M. F. consumed by the impedance of the system is represented by OZ and is the resultant of OR and OX. The generator voltage is the resultant of this E. M. F. OZ and the E. M. F. at the receiving end is thus represented by OE_0, which is larger than OE and larger than $OE + OR$.

In diagram B, Fig 3, is assumed the same current flowing over the same line, but the current is made to lead by a certain angle. It is readily seen in this diagram that the E. M. F. which has to be generated is less than the E. M. F. desired at the receiving end, and of course much less than that in the first mentioned case, where the current was in phase with the voltage at the receiving end.

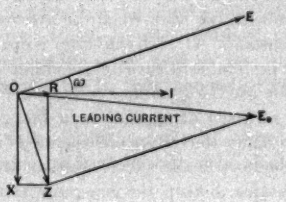
In diagram c Fig. 3, is shown the same condition with lagging current, and demonstrated plainly how the generator E. M. F. is still higher than the generator voltage for non-inductive load.

Since, however, there is considerable difficulty in getting definite numerical instances worked out with any degree of accuracy by means of graphic methods, the analytical treatment is given below:

A



B



C

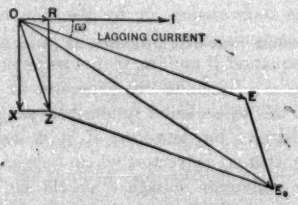


FIG. 3.—Graphic Method of Obtaining the Generator Voltage, e_0, in a Circuit of Resistance and Reactance.

Let e_0 = the generator voltage, the induced E. M. F., if the generator field excitation is kept constant or its terminal E. M. F. if the excitation is varied with the load so that the generator voltage is kept constant.

e = the E. M. F. at collector rings of rotary converters.

$c_1 + j\,c_2$ in symbolic expression the current taken by the rotary, c_1 the energy current, and c_2 the wattless current, and

$z = r - j\,x$ the total impedance between e_0 and e, that is the total impedance of transmission system and generator in the case of a generator with constant excitation, and the impedance of the transmission line[1] if the generator voltage is constant.

We have the following equations:

$$e_0 = e + (c_1 + j\,c_2)\,(r - j\,x) \text{ or} \qquad (A)$$

$$e_0^2 = (e + c_1\,r + c_2\,x)^2 + (c_1\,x - c_2\,r)^2 \qquad (B)$$

At non-inductive load $c_2 = 0$

$$\text{therefore } e_0^2 = (e + c_1\,r)^2 + c_1^2\,x^2 \qquad (C)$$

At no load $c_1 = 0$

$$\text{therefore } e_0^2 = (e + c_2\,x)^2 + c_2^2\,r^2 \qquad (D)$$

Usually it is required that the converter voltage shall remain constant or increase with the load, when the generator voltage or excitation is kept constant. If each converter is operated from its own generator, it is preferable to leave the generator field excitation constant and thereby use the generator reactance for phase control and avoid any necessity of regulation in the generating station. If, however, many converters are operated from one generator, or if all generators in the power station are operated in multiple, it is preferable to keep the generator terminal voltage constant, and control the converter voltage by outside reactances in a low reactance transmission, or by the line reactance in a highly inductive transmission.

In either case equation B applies, if it is borne in mind that the impedance $Z = r - j\,x$ includes the generator in the first case and only refers to the transmission system between generator and converter in the latter. Furthermore, if it is desired to have overcompounding, that is, converters which increase their voltage with the load, the converter voltage e should be expressed by $e = e + p\,c_1$ where e is the no load E. M. F. and $p\,c_1$ gives the increase in E. M. F. at a load of c_1 amperes.

Thus the general equation of phase control becomes

$$e_0 = e + p\,c_1 + (c_1 + j\,c_2)\,(r - j\,x) \text{ or}$$

$$e_0^2 = (e + p\,c_1 + c_1\,r + c_2\,x)^2 + (c_1\,x - c_2\,r)^2 \qquad (E)$$

where $p\,c_1$ is 0 in a rotary with constant voltage at all loads. As seen, the overcompounding enters the equation in essentially the same way as a resistance and can therefore in most problems be assumed as such.

1. Including transformers and other intermediary apparatus.

Looking over equation A, we find that it is the relation of E. M. F.'s and currents in an alternating current transmission line, indeed, the usual equation for determining line drop, etc., and corresponds to equation $e_0 = e + c\,r$ in a direct current line.

Since a converter can be made to take leading or lagging current by changing its field excitation and thus its counter E. M. F., such an installation gives the most favorable opportunity to study the fundamental principles of alternating current.

It may perhaps seem as if it would be rather a delicate operation to accomplish this potential control in commercial circuits where available instruments, etc., would prohibit the determination of leading or lagging currents necessary for the control with any degree of accuracy, yet the problem is perfectly simple and resolves itself to adjusting the shunt excitation for a given running light current which is predetermined by calculation, and afterwards to adjust the series field so that the converter takes minimum current, that is, runs non-inductively at the desired load.

It will then follow that the potential control for all intermediate loads is perfect, at least within very close limits, as I shall show later.

The running light current is readily determined from equations (B) and (C).

Let $c_1 = c_n$ at non-inductive load where $c_2 = 0$

$c_2 = c_0$ running light where $c_1 = 0$

then $e_0 = (e + e_n\,r)^2 + c_n^2\,x^2$ which substituted in equation (C) gives $(e + c_n\,r)^2 + c_n^2\,x^2 = (e + c_0\,x)^2 + c_0^2\,r^2$ which solved in reference to c_0 the running light current gives

$$c_0 = \frac{e\,x}{z^2} + \sqrt{\frac{e^2\,x^2}{z^4} + C_n^2 + \frac{2\,e\,C_n\,r}{z^2}} \qquad (F)$$

denoting the converter voltage $e = 1$

The converter full load current $c_1 = 1$

and assuming the converter non-inductive at full load we get the equation of running light current.

$$c_0 = -\frac{x}{z^2} + \sqrt{\frac{x^2}{z^4} + 1 + \frac{2\,r}{z^2}}$$

The magnitude of this current is calculated and plotted in Fig. 4 for different conditions of resistance and reactance.

Curve *a* shows the conditions at 15% energy loss.
" *b* " " " 10% " "
" *c* " " " 5% " "

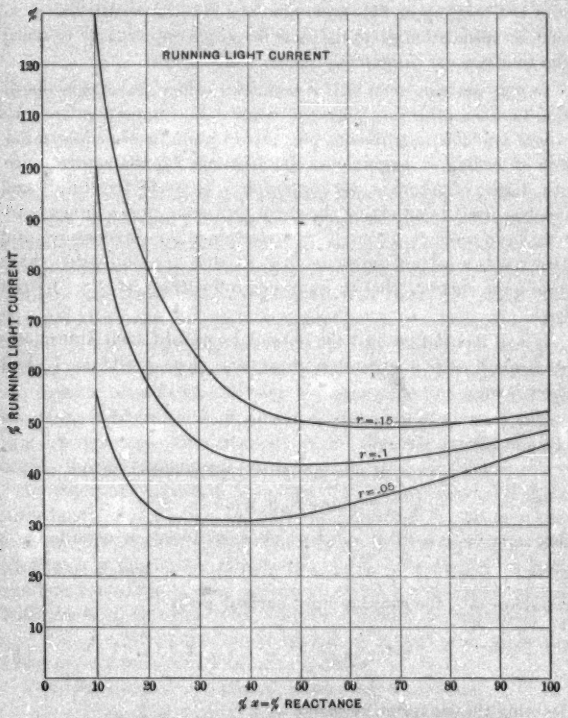


FIG. 4.—Dependence of Running Light Current on the Reactance and Resistance of the Circuit. Converter Adjusted to Run Non-Inductively at Full Load.

It shows plainly that the lesser the loss the lesser is the running light current and the lesser reactance is necessary. In a system of 5% loss 30% reactance, that is such a reactance should be inserted as gives 30% of rated voltage at full load current. In a

system of 15% loss, however, 50% reactance would be about as low as we would go, and yet the running light current is 50% of full load current.

It is evident also that the more energy loss, the higher should the generator voltage be relative to the converter voltage as shown by the equations.

In the above case we assumed that the converter should run non-inductively at full load; usually, however, it is preferable to run it non-inductively at a somewhat lighter load, say $\frac{3}{4}$ load. Under these conditions substituting in equation F for $c\,n = .75$ the equation of the running light current and non-inductive load at $\frac{3}{4}$ load becomes,

$$c_0 = -\frac{x}{z} + \sqrt{\frac{x^2}{z^2} + .56 + \frac{1.5}{z^2}}$$

In this case with 40% reactance, the running light current is only 20% of full load current at 10% energy loss, and with 50% reactance only 34% at 15% energy loss. Therefore it would indeed seem, as if it always were more favorable to run with non-inductive load at $\frac{3}{4}$ load. This is still more evident, when investigating the power factor curves with these adjustments, yet there are some reasons why sometimes converters are adjusted for non-inductive load at full load as will be seen.

The maximum output of a converter is practically unlimited provided the generator voltage can be increased, and there is sufficient capacity behind the generator.

With the best arrangement of reactance, resistance and generator voltage, this is limited only by the resistance in the transmission and is expressed by equation $c_1 = \frac{e_0^2}{4\,e\,r}$ as will be shown later. Usually, however, the maximum output is less than this, so that only the maximum value can be obtained by this equation, so that the above equation should be written $c_1 = \frac{e_0^2}{4\,e\,r}$ where c_1 is the limit of energy current, that is output, e_0 and e the generator and converter E. M. F.'s respectively and r the resistance in the system.

To obtain the maximum possible output with a given line impedance, the converter should run non-inductively at higher outputs as the generator voltage increases, so that for instance, f the system has 10% energy loss and 40% reactance and the

generator voltage is kept 12½% above the converter voltage, the maximum output is 2.12 times rated output and the converter should run non-inductively at ⅝ of rated load. If the generator voltage were 18% higher than the converter voltage, the output would have been 2¼ times the rated, and the converter would run non-inductive at full load and finally, if the generator voltage were 30% higher than the converter voltage the maximum output would be. 2.55 times the normal output but the non-inductive load would have been 1½ times full load. In other words the converter should run with lagging current up to 50% overload.

In commercial installations, however, where there is always a limit to the generator voltage, which limit is caused by saturation of the machine, or outside conditions which do not permit of excessive voltage in the power station, the output of a converter is by no means unlimited, and thus some care has to be taken, to get the proper reactance, since with a given difference in generator and converter potential, the output changes as we change the reactance and resistance.

Referring again to equation (B).

By solving the equation in reference to c_2, it is

$$c^2 = -\frac{e\,x}{z^2} + \sqrt{\frac{e^2\,x^2}{z^4} - c_1^2 - \frac{2\,e\,c_1\,r}{z^2} + \frac{e_0^2 - e^2}{z^2}}$$

c_2 becomes maximum for

$$\frac{e^2\,x^2}{z^4} - c_1^2 - \frac{2\,e\,c_1\,r}{z^2} + e_0^2 - \frac{e^2}{z^2} = 0$$

or

$$c_1 = -\frac{e\,r}{z^2} + \sqrt{\frac{e^2\,r^2}{z^4} + \frac{e^2\,x^2}{z^4} + \frac{e_0^2 - e^2}{z^2}}$$

where c_1 is the limiting current which directly represents the limit in output.

Solving this equation in reference to z the impedance, the following expression gives the impedance corresponding to a given maximum output:

$$Z = +\frac{1}{c_1\sqrt{2}}\sqrt{e_0^2 - 2\,e\,r\,c_1 \sqrt{e_0^4 - 4\,c_1\,r\,e\,e_0^2}}$$

for maximum output

$$e_0^4 = 4\,c_1\,r\,e\,e_0^2 \text{ or } c_1 = \frac{e_0^2}{4\,e\,r}$$

as given above.

Inserting this value we get,

$$Z = \frac{e_0}{2\,c_1}$$

or if

$$\frac{e_0}{e} = k,\ z = \frac{2\,r}{k}$$

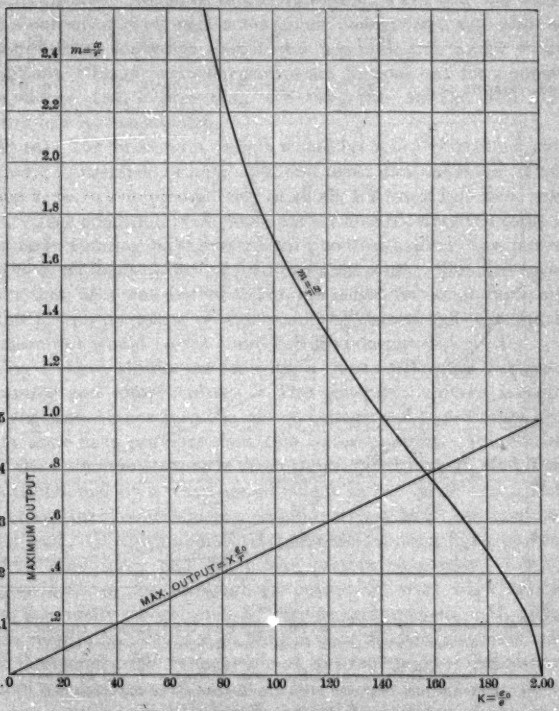


FIG. 5.—Curve Giving Maximum Possible Output and Corresponding Ratio of Reactance and Resistance for Given Ratios of Generator and Converter Voltages.

and

$$x = \frac{r}{k}\sqrt{4 - k^2}$$

or to get maximum output for a given ratio of generator and

converter voltage the ratio of reactance and resistance is fixed by equation

$$\frac{x}{r} = \sqrt{\frac{4 - k^2}{k}}$$

thus

$\frac{e_0}{e} =$	1	1.1	1.15	1.2
$m = \frac{x}{r} =$	1.73	1.52	1.42	1.33
max. output =	$.25\frac{e_0}{r}$	$2.75\frac{e_0}{r}$	$.2875\frac{e_0}{r}$	$.302\frac{e_0}{r}$

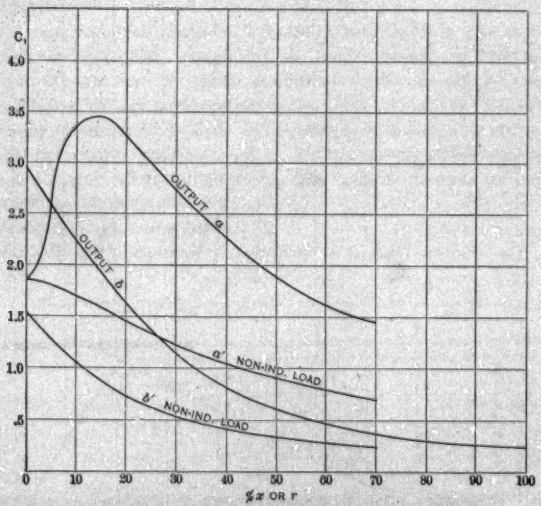


FIG. 6.—a and a' are Curves Showing How the Maximum Output of a Converter Depends Upon the Reactance in the System. Full Load Assumed to be 1. Energy Loss 10%. Generator Voltage 19% Higher than Converter Voltage. b and b' show how Output Depends upon the Resistance at 40% Reactance.

$E_0^2 = 1.4 \quad E = 1.$

$r = .1$ and Variable x = curve a.

$x = 4$ and Variable r = curve b.

Curve a' = Position of Non-Inductive Load Variable x.

Curve b' = Position of Non-Inductive Load Variable r.

It must, however, be borne in mind that this limiting value by no means represents the best "all around" condition. Indeed by choosing the reactances from results obtained from this equation

and Fig. 5 the power factor of the system would in general be very poor, therefore it would be better either to sacrifice some in output and increase the reactance or raise the generator voltage some and thus keep the output with the greater reactance.

In Fig. 6 a couple of curves a and a_1 are given showing the relation between reactance in the transmission line and the maximum output of a converter for given energy loss, and also at which load the converter should run non-inductive under those conditions. These particular curves are figured out for a generator voltage 19% higher than the converter voltage and 10% energy loss in transmission.

As seen, the maximum output would be 3.45 times rated output with a reactance of only 15% and with the converter so adjusted as to run non-inductively at about 1.6 times full load current. This condition is of course rather absurd, since it means a very large running light current with consequent very low power factors at all loads up to full load or thereabouts. The curve also shows that if it was desired to run the converter non-inductively at full load, a reactance of 42% should be chosen and the maximum output would be 2.2 times full load output.

Fig. 7 shows similar curves with a lesser difference between generator and rotary voltage. The generator voltage is kept ten per cent. higher than the rotary voltage and the energy loss is the same as in previous case, that is, ten per cent. It is shown that the maximum output is three times rated output, and that this is obtained by a reactance of about 15 per cent. Also that the converter then should run non-inductively at 95 per cent. of rated load. With this small reactance the running light current will be quite large and the power factors comparably low at lighter loads, so that although the power factor is high at full load, it is really not as good a feasible arrangement. If, under these conditions, that is with ten per cent. higher generator voltage than converter voltage, it was desired to get a maximum output of double rated output of the converter, a reactance of 42 per cent. would be suitable, and the converter should run non-inductively at 65 per cent. of full load, which gives very good general characteristics as far as power factors are concerned.

The output of the converter depends also necessarily upon the resistance in the line. To illustrate this another set of curves is plotted in Fig. 6 denoted by b and b_1, b showing the

maximum output for different values of energy loss with a reactance in the system of 40 per cent., and b_1 at which load the converter should run non-inductively under those conditions.

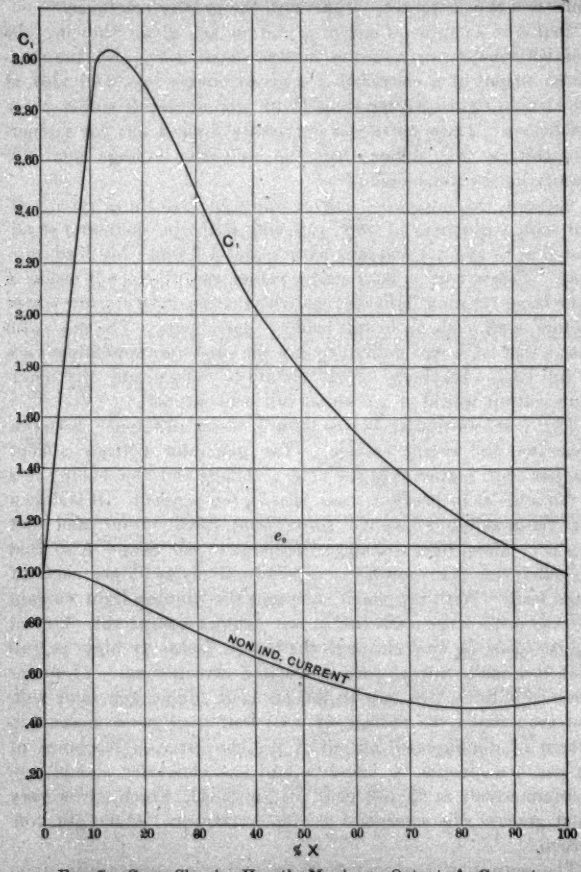


FIG. 7.—Curve Showing How the Maximum Output of a Converter Depends Upon the Reactance in the System. 10% Energy Loss, Generator Voltage 10% Higher than Converter Voltage, and Full Load Output of Converter Assumed as 1.

As seen, the output increases very rapidly as the energy loss decreases at least within commercial conditions, that is between five and 20 per cent. loss. The maximum output with 40 per cent. reactance and zero resistance would be three times rated output.

With five per cent. energy loss the maximum output would be 2.6 times rated output, and the converter should be adjusted for non-inductive load at $1\frac{1}{4}$ full load.

The condition which is to be met with in a commercial installation is, that the generator is limited to a certain value,

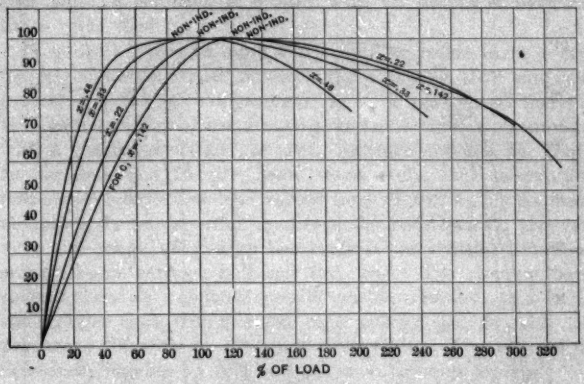


FIG. 8.— $E_0 = 1.15$ $E' = 1.$ $r = .1$ x variable.
Power Factors for various Reactances. Maximum Output for $x = .142$ is 33% of full load.
Power Factor Curves of Converters Operated from Generator with 15% Higher Voltage than the Converter Voltage. Energy Loss 10%.

that the converter voltage shall be maintained constant at all loads or increase with the load, and that that there shall be a certain margin in maximum output and the highest power factors at all loads.

As an illustration, in Fig. 8 is plotted a number of curves giving the power factors at the converter under the assumption that the generator voltage is constant and 15 per cent. higher than the converter voltage, that the full load E. M. F. and current is unity, and the resistance or rather the energy loss is ten per cent. for different values of reactance.

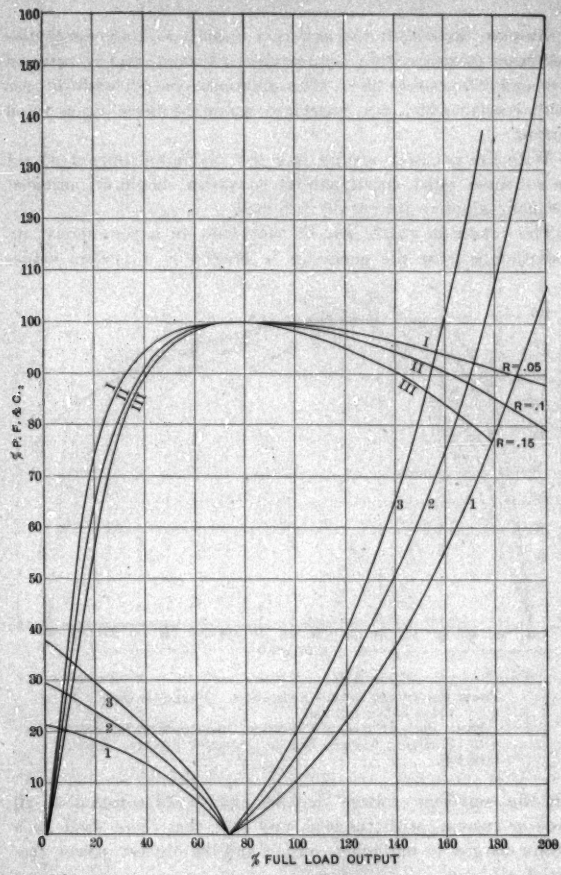


FIG. 9.—

$e = .4$, $r = \begin{cases} .05 & \text{Curve I.} \\ .10 & \text{“ II.} \\ .15 & \text{“ III.} \end{cases}$

$e = 1$ Non-ind. at ½ load.

Wattless Current given $\begin{cases} \text{Curve 1.} \\ \text{“ 2.} \\ \text{“ 3.} \end{cases}$

Curves Showing Power Factors of Converters adjusted to run non-inductively at ½ load on a Circuit of 40% Reactance, and 5, 10 and 15% Energy Loss, and Constant Voltage at Converter at all loads.

As can be ascertained by equations given above, the maximum output that could be obtained under such circumstances is 3.3 times rated output with a reactance of 14.7 per cent. The power factor under these conditions is as seen, exceedingly unsatisfactory. The converter would have to run non-inductive at 140 per cent. load, and the power factor at $\frac{1}{4}$ load is 20 per cent., $\frac{1}{2}$ load 60 per cent., at $\frac{3}{4}$ load 82 per cent. Apparently, judging from the power factor curve the most satisfactory reactance would be about 46 per cent. The converter is practically non-inductive at full load, has 95 per cent. power factor at $\frac{1}{2}$ load and 71 per cent. at $\frac{1}{4}$ load with an output of over 2.5 times rated load.

In general it may be quite satisfactory to decide upon a certain reactance and load at which the oonverter shall run non-inductively, and let the generator E. M. F. correspond thereto. In Fig. 9 are plotted some curves under a similar assumption with a reactance of 40 per cent., resistance of five per cent., 10 per cent. and 15 per cent. respectively, and assumption that the non-inductive load should be $\frac{3}{4}$ of full load. As seen, the power factor is better the lower the resistance is, yet even over this range of energy loss there is very little difference around loads between half and full load. The running light current, however, depends greatly on the line loss, and is, as seen, 20 per cent. with five per cent. energy loss, 30 with 10 per cent., 38 with 15 per cent. energy loss. The current is necessarily lagging up to $\frac{3}{4}$ load and then becomes leading.

If the converter were to be over-compounded the power factors would have been somewhat poorer, since as stated above, over-compounding acts in essentially the same way as increased energy loss.

This is illustrated in Fig. 10 where corresponding curves are plotted for a converter of ten per cent over-compounding, that is, for a converter where the terminal voltage is expressed by $e = e + p\ e_1$.

The power factor at the generator will be slightly poorer at light load and slightly better at heavy load than at the converter, assuming that the transmission line has considerable self-induction.

In Fig. 11 is plotted in a different way the wattless current in the line corresponding to condition given in Fig. 9. Here it is plainly seen how the amount of lagging or leading current is not

proportional to the load, consequently the magnetizing or demagnetizing effect of the armature current not proportional thereto.

In practice the shunt field is so adjusted that the converter takes the required lagging current running light, and the series

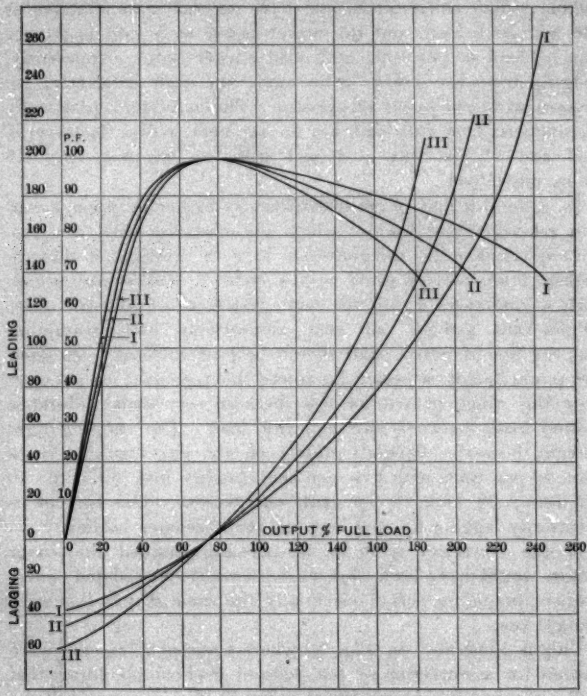


FIG. 10.— $x = .4$, $r = \begin{cases} .05 \text{ Curve I.} \\ .10 \quad \text{“ II.} \\ .15 \quad \text{“ III.} \end{cases}$

$e = 1 + .1\ \mathcal{O}$, non-ind. at ½ load.

Curves giving Power Factors of Converter. Reactance is Assumed as 40%, Energy Loss, 5, 10 and 15% respectively. Converter Runs Non-inductively at ½ load, and over compounded 10%.

field is made to increase the excitation so as to give the required field strength at non-inductive load. Since, however, the series excitation is proportional to the load, it is evident that without

hand adjustment, it would be theoretically impossible to accomplish perfect phase control; or rather, it would be impossible to have constant voltage at the rotary converter with constant voltage at the generator. The difference, however, is exceedingly small, the series field being slightly too strong at light loads and too weak at overload, in other words the voltage will be slightly

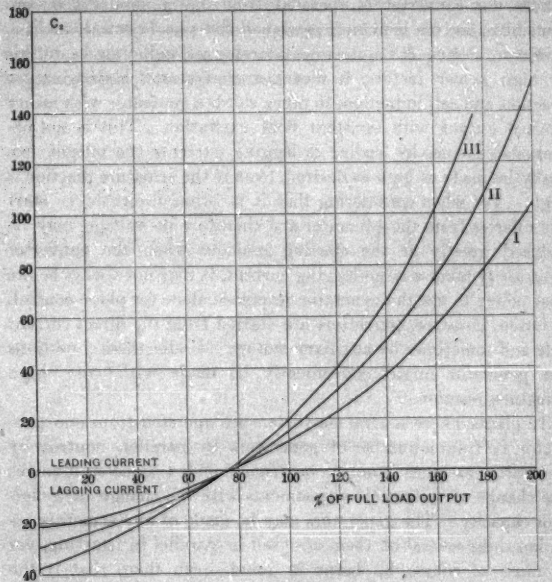


FIG. 11.— $x = .4$, $r = \begin{cases} .05 & \text{Curve I.} \\ .10 & \text{" II.} \\ .15 & \text{" III.} \end{cases}$

Curve Showing amount of Leading and Lagging Current in Line in order to get perfect Phase Control. 40% Reactance. 5, 10 and 15% Energy Loss.

increased at the converter up to non-inductive load and will drop slightly at overloads.

In the preceding we have not considered this point and have assumed that the theoretical loading or lagging current can be obtained at any load. Consequently the values which we have given of maximum output of the converter are not quite true in

practice but the output is always less. The amount of this decrease in output depends upon the magnetic characteristic of the machine and can readily be determined in practice.

Before closing the paper it might be well to discuss the conditions which govern the amount of reactance, etc., in the machines. As stated in the beginning of the paper, in plants where one converter is operated from one generator it is convenient to use the generator reactance for phase control. Since, however, as seen, it requires considerable self-induction to do this at high power factors, it means a generator of high armature reaction and self-induction, in other words a generator with rather limited output with constant field excitation. This is not objectionable, since by leading or lagging currents the output can really be made as high as desired, even if the armature reaction is high. Yet when considering that it is often desirable to start the rotaries from the generator and therefore its voltage may be reduced greatly at the starting moment when the converter necessarily takes a large lagging current, it may not always be the best policy to use the generator reactance alone for phase control.

Often, however, converters are started from the direct current side and sometimes by auxiliary motors. Under these conditions the generator might conveniently be made with very high armature reaction.

In plants where several converters are operated from one generator, or from a number of generators in parallel, control by generator reactance is not to be recommended, since it involves the change of series field adjustments with the change of generator capacity. The generators may be made of any armature reaction since several of them are used in parallel in starting, yet it must of course be borne in mind even then, that if the armature reaction is too high, the large current taken in starting a converter from the alternating current side, may cause the voltage to drop so far as to seriously disturb the system in general.

Regarding converters themselves, it is obvious from the preceding discussion that they must be of composite type, that is, they must have shunt and series excitation.

Shunt wound converters do not permit of automatic phase control, but at constant adjustment of the field will always run at the same wattless current at all loads, so that, for instance, if the field is adjusted for non-inductive load at one load, the

converter will run non-inductively at all loads, and therefore the drop in voltage at full load will correspond to the energy loss in the system, and the converter voltage will be higher at lighter loads than at heavier loads, therefore will vary as the load varies, and phase control is thus feasible only by hand regulation, that is, in systems where the load varies only slowly as in lighting circuits.

Reaction converters take lagging currents at all loads and thus are in general objectionable, the more so as they do not allow any automatic potential control.

A Paper to be presented at the 15th General Meeting of the American Institute of Electrical Engineers, Omaha, June 29, 1898.

THE COMMUTATED CURRENT WAVE OF A COMPOSITE-WOUND ALTERNATOR.

BY DUGALD C. JACKSON.

The accompanying curves were taken by Mr. H. S. Webb, one of my graduate students, during a relative study of the current and pressure waves in the various branches of a 50 kilo-

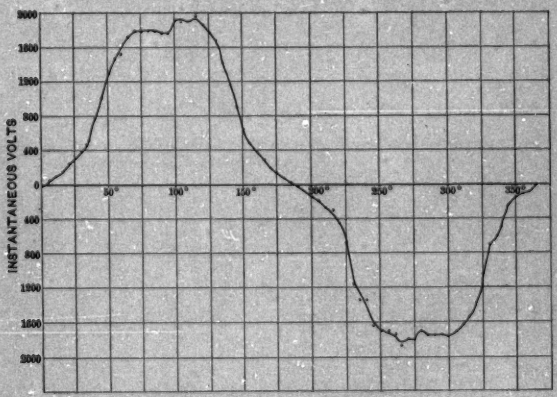


FIG. 1.—Open Circuit Pressure Wave.

watt alternator with composite winding. The normal pressure of the alternator is 1,100 volts, and the frequency is 125 periods per second. The machine was built several years ago by the Thomson-Houston Electric Co., and has the usual construction

of the old belt-driven alternators with ten field-poles and a toothed core, ribbon-wound armature. The driving speed dur-

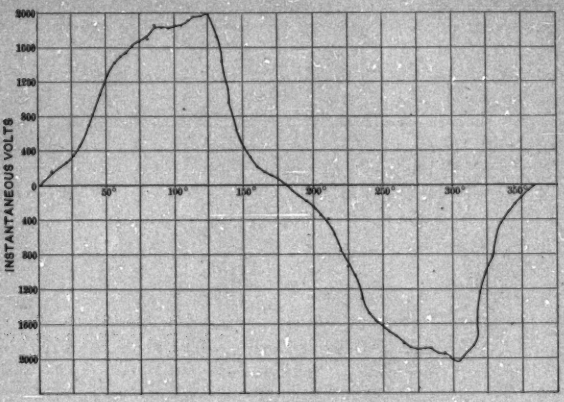


FIG. 2.—Pressure Wave with Load of 22 Amperes.

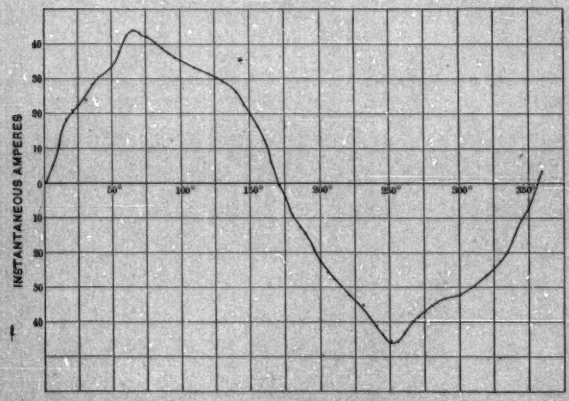


FIG. 3.—External Current Wave with Load of 24 Amperes.

ing the tests was quite unsteady, as the machine was driven by the same engine that drove two street railway generators: and this resulted in irregularities which are evident in the curves.

I am not acquainted with any results which have heretofore been given in the TRANSACTIONS of the INSTITUTE or elsewhere, which show the characteristic form of the commutated current waves in the series field of an alternator, and I therefore present these curves. The question of the form of the commutated

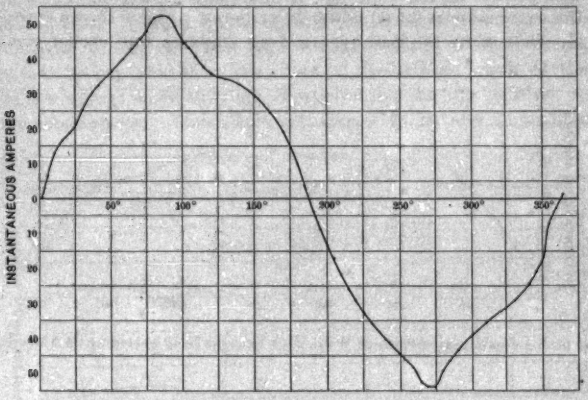


FIG. 4.—External Current Wave with Load of 28 Amperes.

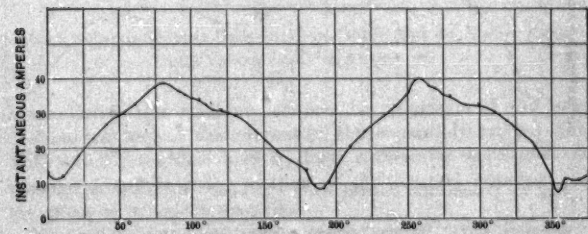


FIG. 5.—Commutated Current Wave with Load of 22 Amperes.

current wave is mentioned in two or three text-books, but experimentally determined waves do not appear.

In the series of curves presented herewith, Fig. 1 is the pressure wave at open circuit, and Fig. 2 is the pressure wave when the armature current equalled 22 amperes. A comparison of

these gives an idea of the extent of the effect of armature reactions on the form of the wave. Figs. 3 and 4 respectively, show the waves of external current when the load amounted to 24 and 28 amperes, the machine being worked upon an incandescent lamp load through partially loaded transformers. Fig. 5 shows the wave of commutated current in the series field when the external current equalled 22 amperes. Fig. 6 shows a similar wave when the load averaged 24 amperes but varied some-

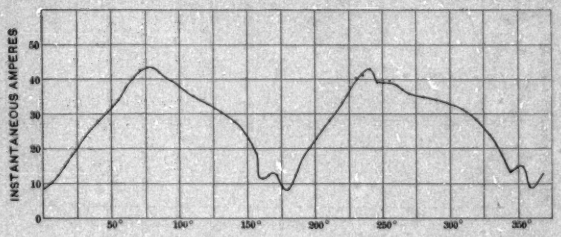


FIG. 6.—Commutated Current Wave with Variable Load averaging 24 Amperes.

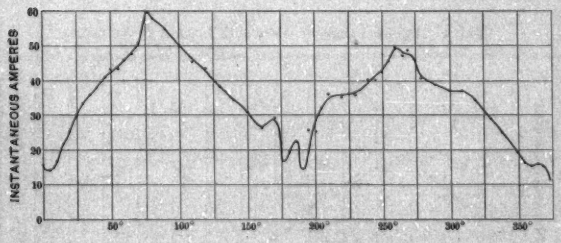


FIG. 7.—Commutated Current Wave with Load Falling from 28 Amperes.

what, and Fig. 7 shows a similar wave during a period in which the load on the machine was dropping rapidly. At the beginning of this period the load was 28 amperes. Fig. 8 shows the pulsation of current set up in the circuit of the continuous current external exciter, by the pulsations of the magnetism in the fields of the alternator when the load is respectively 23, 25 and 28 amperes.

A comparison of the external current waves, such as are shown in Figs. 3 and 4, with the commutated waves such as are shown in Figs 5 and 6, shows that the commutated waves have practically the same form as the original waves of current (with alternate loops inverted) except in regions occupying about 25 degrees on either side of the zero points of the original waves. Within these regions the commutated waves do not come to zero and they take a characteristic toothed shape, as is to be expected from the effect of the self-induction of the field windings when they are short-circuited by the brushes on the commutator. There does not appear to be any immediate

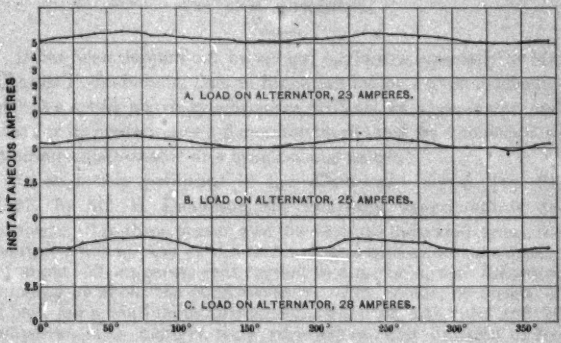


FIG. 8.—Curves Showing Pulsation of Current in Separate Exciter Circuit with Various Loads on Alternators.

connection in this machine between the pulsations of the series field current and of the current in the separately excited field. The latter pulsations are, in fact, sufficiently accounted for through the effect of the rotation of the toothed armature and the effect of armature reactions.

The points on these curves were located by a balance or potentiometer method similar to that described on pages 296 and 297 of Jackson's "Alternating Currents," but, instead of the usual galvanometer or telephone receiver, Mr. Burgess' portable form of capillary electrometer (which is described by him in a paper presented to this meeting) was used to indicate the point of balance. This instru-

ment has been used in tracing alternating current curves by quite a number of my students, and it is the unanimous verdict that the instrument greatly increases the convenience, the accuracy, and the speed of the method. We have had a number of the instruments in use during the past winter and have found them perfectly portable, difficult to injure, and practically unaffected by external inflences.

A Paper to be presented at the 15th General Meeting of the American Institute of Electrical Engineers, Omaha, June 29, 1898.

[ADVANCE COPY.—SUBJECT TO REVISION.]

SOME TESTS WITH AN INDUCTION GENERATOR.

BY A. F. MC KISSICK.

It has been pointed out by several engineers, especially by Mr. Charles P. Steinmetz, that an induction motor, driven mechanically at a speed above synchronism, will act as a generator, and that for its operation as a generator there must be connected to it either an alternator or a synchronous motor.

In an article published in the *Electrical World*, Jan. 21, 1893, by Mr. E. Danielson, the following experiment is described: "A three-phaser A of 50 volts (as measured from the neutral point to each of the brushes), and with a current capacity of about 60 amperes, sent current to a motor B, with inductive winding of about the same capacity. The motor, by means of a belt, ran a continuous current dynamo, C of 330 volts and 16 amperes. The matters were reversed so that dynamo C, fed from a 330-volt circuit, ran the motor B as a generator, sending a rotary current to the three-phaser A, which then became a motor. The belt was then thrown off of A, and trials were made to break it out of step with the current, but without success."

Again in the excellent paper of Mr. Charles P. Steinmetz on "Induction Motors" presented before the AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS on July 26, 1897, the use of an induction motor as a generator was referred to as follows: "As generator the induction machine differs from the synchronous alternating current generator, or generator with constantly excited field, in so far as the latter can yield current and output at any power factor, that is, any phase displacement corresponding to the load, while in the induction generator at given terminal voltage to every value of current output a certain power factor of load corresponds. That is, to derive a certain value of current from the induction generator, the total load put on it

must have the particular power factor corresponding to this current, and besides leading current or if the power factor of the load changes, current and voltage of the induction generator will change accordingly. In consequence thereof, in general the induction generator is stable only, if at least a part of the load consists of synchronous motors."

In the discussion that followed this paper, Mr. Steinmetz speaks further of the induction generator as follows: "The induction generator is a very important piece of apparatus, I believe. It is, however, somewhat restricted in its application, due to the necessity of having as load a circuit of leading current. But whenever the conditions are such that it can be used, as for operating synchronous motors or rotary converters, this type of machine has the great advantage of the absolute absence of continuous current exciting circuits, collector rings, or any other parts requiring attention. The voltage is generated in a stationary structure and the revolving part is a solid structure of iron and copper bars. Furthermore, as soon as the circuit is opened or short-circuited, the power is gone and the machine is dead, so that you get here a type of alternator requiring no attention whatever. Besides you can run it at different speeds and still get the same frequency out of it. There is another interesting feature noticeable when comparing the induction motor curves and the induction generator curves. The same machine as induction generator gives a considerably larger output electrically than as induction motor mechanically. In a future paper I shall dwell more particularly on the induction generator, and may mention here only that I have operated synchronous motors from an induction generator, the mechanical output from the synchronous motor driven by the induction generator was larger than the maximum mechanical output which could be derived from the same induction machine as induction motor."

The results of the tests made, show close agreement with the theory as given by Mr. Steinmetz.

Apparatus.

The tests described in this paper were made with a General Electric 5-H.P., 3-phase 220-volt, 6-pole, 60-cycle induction motor (with starting resistance in secondary rotating element), driven by a 10-H.P. 110-volt Thomson-Houston direct current motor.

The machine used as an alternator and synchronous motor in the tests was an "Ideal" 5-kilowatt 3-phase, 60-cycle, 10-pole, 220-volt machine, and could be belted either to a jack-shaft or to a 3-kilowatt-Edison-125-volt dynamo.

The curves in Fig. 1 show the operation of the induction machine as a motor with varying output.

PART I.

In the tests under this head, the belt connecting the Ideal alternator to the jack-shaft was in each case thrown off after starting and the alternator run as a synchronous motor, either empty or driving the 3 kilo-watt-Edison dynamo.

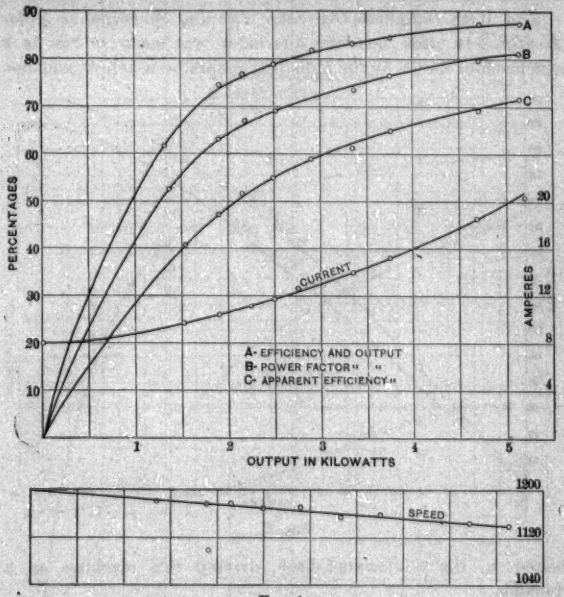


FIG. 1.

Curve of Magnetization.—The method of making this test is as follows: Start induction motor from Ideal alternator, the 10-H. P. direct current motor being belted to the induction machine when up to speed, supply current to 10-H. P. direct current motor, driving induction machine near synchronism, and throw off belt connecting alternator to jack-shaft. This alternator will then run as a synchronous motor, receiving its current from the induction machine, which is now being driven as a generator by the 10-H. P. direct current motor belted to it.

By varying the exciting field current of the synchronous motor, the voltage of the induction generator is varied, and from the readings of field current in synchronous motor and voltage at terminals of induction generator, the curve, marked "Induction generator" in Fig. 2 is obtained.

The magnetization of the Ideal alternator is also given in Fig. 2, which is almost identical with the magnetization curve of the induction generator.

Test With Induction Generator Driving Synchronous Motor Loaded.—In this test the alternator was made to run as a synchronous motor by the induction generator as before, and then

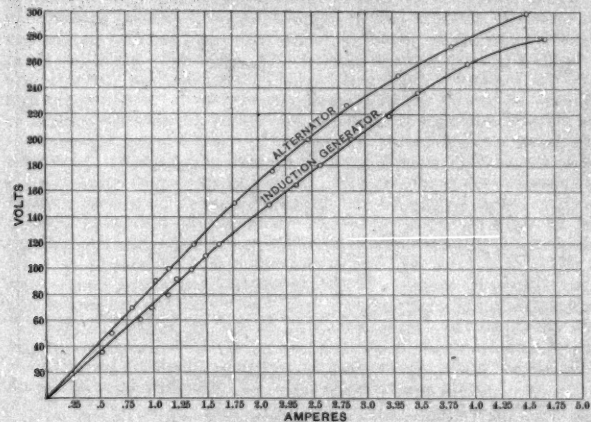


FIG. 2.

belted to the 3-kilowatt-Edison, driving this machine as a dynamo.

The results of this test are shown in Fig. 3.

In making the calculations for efficiency, the energy required for the excitation of the synchronous motor fields was charged up against the synchronous motor.

Fig. 4 gives the results of the following test: With the connections the same as in the preceding test, and the load on the 3 kilowatt Edison maintained constant, the voltage of the induction generator was varied, and corresponding current, watt and speed readings recorded.

The voltage was varied from 260 to 125, the synchronous motor falling out of step on attempting to lower the voltage to 100.

Curve A gives the variation of supply current with field current, curve B variation of supply current with voltage of supply, and curve C is plotted between power factor and voltage of supply.

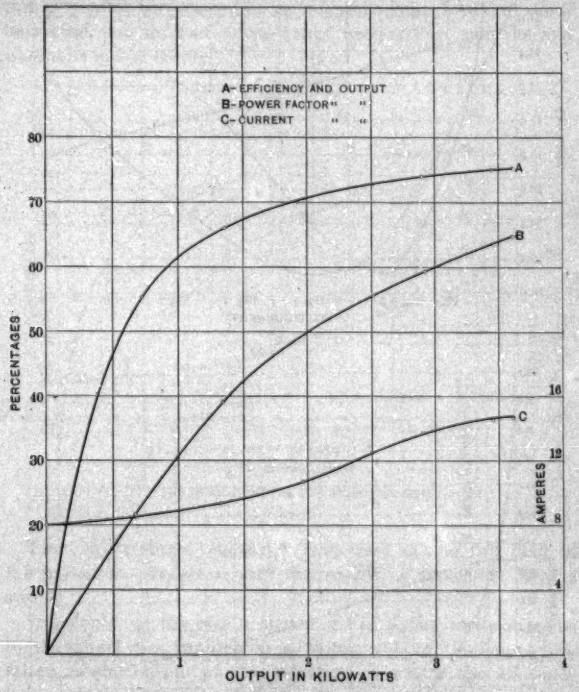


FIG. 3.

It is interesting to note the similarity between the general shape of curve A and the curve plotted betweent he same co-ordinate values as curve A, obtained from a synchronous motor with constant load, supplied with alternating current at a

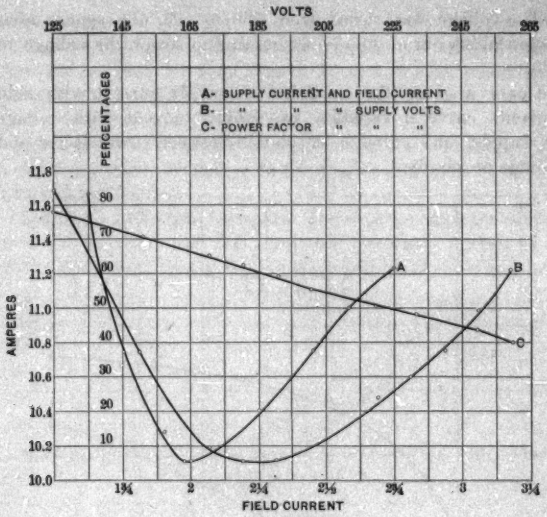

VOLTS
125 145 165 185 205 225 245 265
PERCENTAGES
A- SUPPLY CURRENT AND FIELD CURRENT
B- " " " SUPPLY VOLTS
C- POWER FACTOR " " "
AMPERES
11.8 11.6 11.4 11.2 11.0 10.8 10.6 10.4 10.2 10.0
80 70 60 50 40 30 20 10
A
B
C
1¼ 2 2¼ 2½ 2¾ 3 3¼
FIELD CURRENT

FIG. 4.

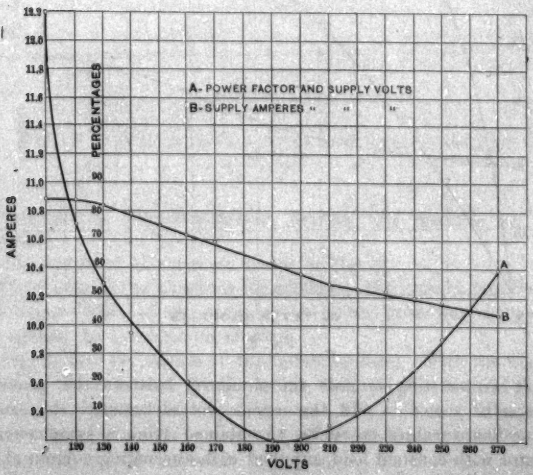

PERCENTAGES
A- POWER FACTOR AND SUPPLY VOLTS
B- SUPPLY AMPERES " " "
AMPERES
11.8 11.6 11.4 11.2 11.0 10.8 10.6 10.4 10.2 10.0 9.8 9.6 9.4
90 80 70 60 50 40 30 20 10
A
B
120 130 140 150 160 170 180 190 200 210 220 230 240 250 260 270
VOLTS

FIG. 5.

constant pressure, and also the similarity between the curves in Fig. 4 and those in Fig. 5, which were taken with the induction machine run as a motor, the load being kept constant on the motor, but the supply voltage varied.

It will also be noted that in both tests the power factor increased uniformly with decrease of voltage.

Test of Induction Generator with Lamp Load.—The induction generator was started as before, and a lamp load that could be varied, put on, it of course being necessary to run the synchronous motor empty.

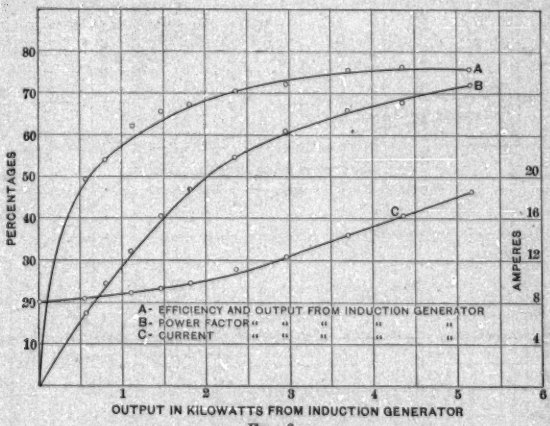


FIG. 6

The load in lamps was varied from zero to the full load of the induction generator, with the results as shown in Figs. 6 and 7.

In calculating the results shown in Fig. 6, the watts supplied to the alternator running as a motor, and to the lamps, was taken as the output of the induction generator, the efficiency being this output divided by the input to the induction generator, and the power factor, this output divided by the total apparent watts from the induction generator.

In Fig. 7 the fact that the energy delivered to the alternator, and that required for the excitation of the alternator fields, does no useful work, is taken into account. So in calculating

the results in Fig. 7, the watts supplied to the lamps were considered as the output, the efficiency being this output divided by

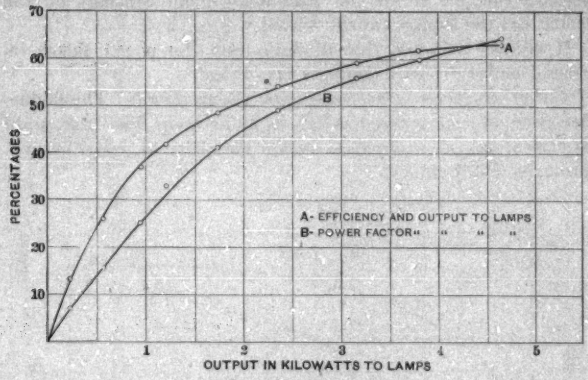


FIG. 7.

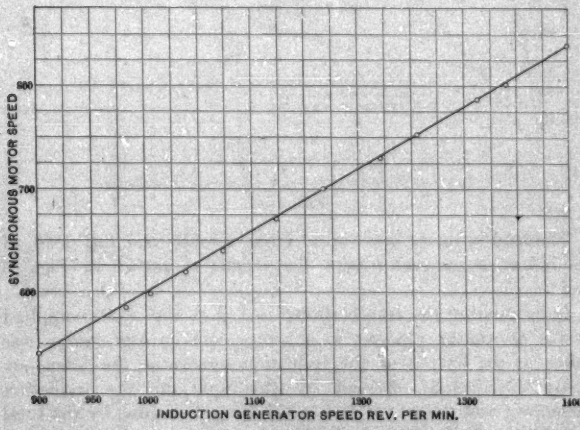


FIG. 8.

the input, and the power factor this output divided by the total apparent watts from the induction generator.

Synchronous Operation of Alternator.—The speed of the induction generator was varied, the voltage of supply being maintained constant, and corresponding readings of speed taken on alternator running as a synchronous motor, which showed that motor ran in synchronism with induction generator. The results are shown in Fig. 8, the full line representing the synchronous speed line, the small circles the points found from readings.

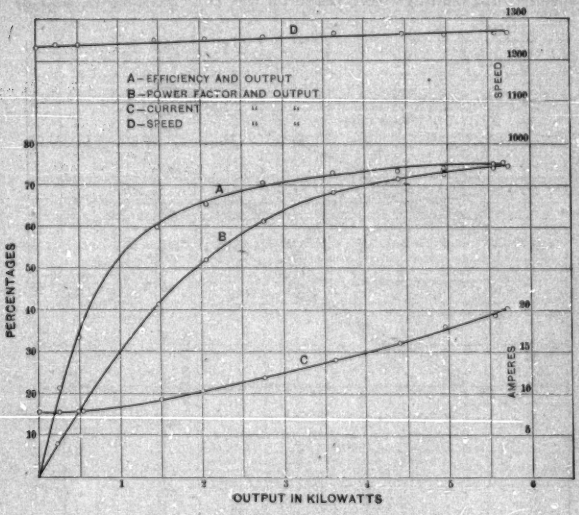


Fig. 9.

In this test, and all preceding tests, the belt connecting alternator to jack-shaft was thrown off, after the starting up of the induction generator.

Part II.

In the tests under this head, the belt connecting the Ideal alternator to the jack-shaft was not removed.

Test of Induction Generator, furnishing current to alternator.—The induction generator was started up as usual, and its speed gradually increased from below synchronism to about six

per cent. above synchronism, readings taken as usual. The alternator was therefore run as a motor and helped to drive the jack-shaft.

The results are plotted in Fig. 9, which will be found by comparison to resemble the curves constructed by Mr. Steinmetz.[1]

It is interesting to note in connection with this test the readings on the two wattmeters, as shown in Fig. 10.

At the beginning of the test before the induction motor was speeded up, readings were taken on the two wattmeters, and as

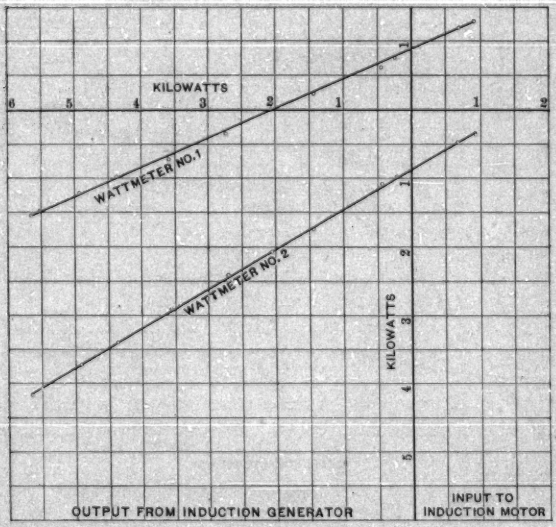


FIG. 10.

the power factor was below 50 per cent., the reading on one wattmeter (No. 2) was negative, hence its connections had to be reversed, and while its connections were reversed, the difference of the two wattmeter readings represented the total true watts. As soon as current was turned on the shunt motor (insufficient however to make this machine act as a motor and

1. TRANSACTIONS, vol. xiv., p. 208.

drive the induction machine), the load on the induction motor was decreased, the power factor decreased, and the corresponding wattmeter readings are shown in Fig. 10.

On furnishing sufficient potential to the shunt motor to make it act as a motor, and drive the induction motor as a generator, the readings on wattmeter No. 1 diminished and finally became negative (necessitating a changing of its connections), while the readings on wattmeter No. 2 continue to increase as shown, and

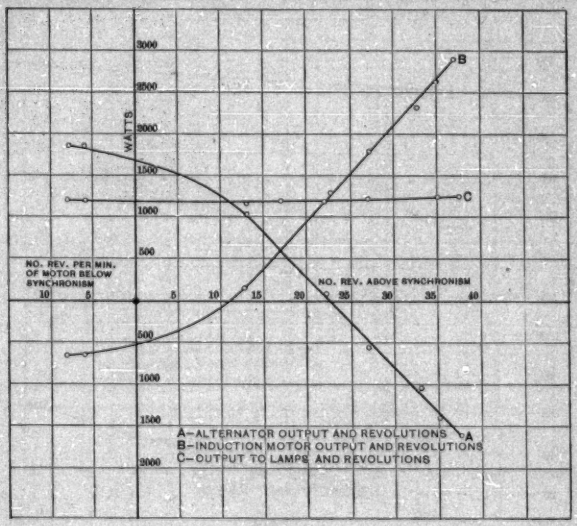


FIG. 11.

always negative. Under these conditions, the total watts will be the sum of the two wattmeter readings, and as the sum is negative, the induction generator is supplying energy to the Ideal alternator.

Test with Induction Generator furnishing Current to Ideal Alternator and to a bank of lamps.—This test is the same as the preceding one, except that a bank of lamps is connected to the three-phase terminals. At the beginning of the test, the alternator furnished current to both induction motor and lamps,

when on speeding up, the induction motor, now running as a generator, supplied current to both lamps and alternator, which acts as a motor helping to drive the jack-shaft.

The results of these tests are shown in Fig. 11.

Curve B is plotted with induction motor output as ordinates and number of revolutions per minute above synchronism as abscissæ. At first the output is negative, which indicates that the motor is receiving energy from the alternator; at 11 revolu-

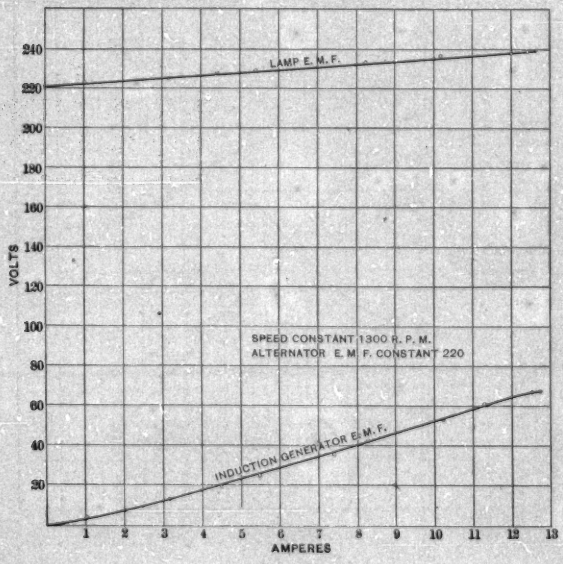


FIG. 12.

tions per minute above synchronism it commences to act as a generator, and at 22 revolutions it is furnishing energy to both alternator and lamps.

As pointed out by Mr. Steinmetz, the induction generator can be run at different speeds and still the same frequency obtained from it. For proof of this, a small induction motor was connected in the above test to the three-phase mains and run with

no load. Variation of the speed of the induction generator from synchronism to 20 per cent. above synchronism, did not in any way affect the speed of the little induction motor, its speed remaining constant, which was almost synchronous with the Ideal alternator.

Of course the above will not hold true when the conditions are as described in Fig, 8.

On attempting to make some tests with this induction generator as a single-phase instead of a three-phase machine, it was

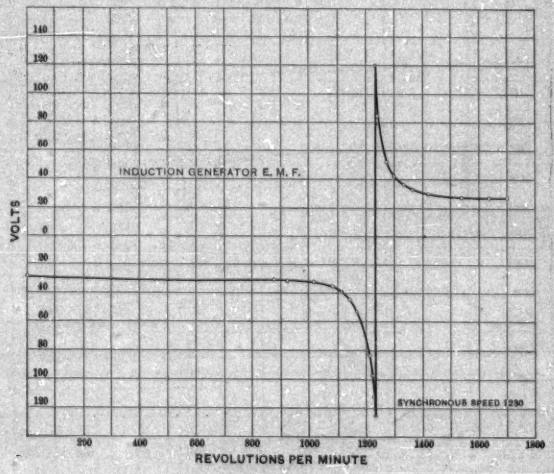


FIG. 13.

found that the current required by the motor when running as a synchronous motor (with belt connecting alternator to jack-shaft thrown off) was excessive, so that this test had to be abandoned.

Uses of Induction Generator.—The most probable use of the induction generator will be for boosting. Mr. Kelly has discovered (*Electrical World*, March 20, 1897), that an induction generator, having its field coils in series with the mains, will act as a booster or feeder regulator, and that this action as a booster at any particular speed above synchronism, will depend upon the amount of current supplied to the field magnets.

In order to use an induction generator as a booster, the fields are connected in series with the main line, and the induction generator driven above synchronism. With no current flowing, there will be no boosting action, but as soon as current flows, the boosting action commences, its amount depending upon the current flowing in the mains, and this amount for a given current may be varied by varying the speed of the induction generator. So that by the use of an induction generator as a booster, we get a machine with no brushes, collector rings or commutator.

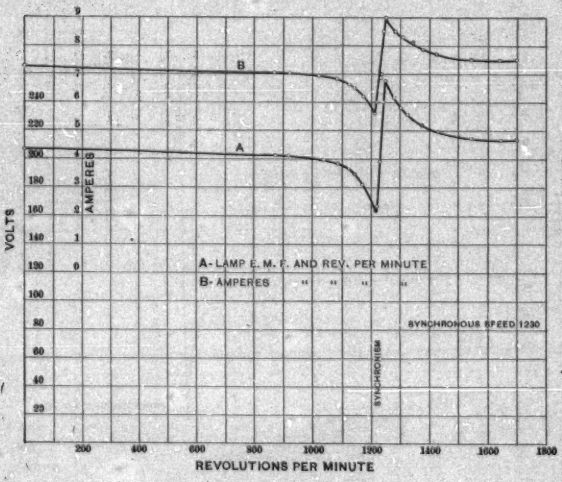


FIG. 14.

In case the induction motor should ever be used for the propulsion of street cars, its action as an induction generator, when driven above synchronism, could be taken advantage of in descending grades, thus pumping back energy into the line, doing away with braking to a great extent.

Test of Induction Generator as a booster.—The induction generator and alternator, used in the above experiments, were connected in series as single-phase machines, with a bank of lamps and readings taken to determine its boosting action.

In the first test as a booster, the speed was maintained constant at 1,300 revolutions per minute (synchronous speed 1,230) and the current flowing in the mains varied from zero to the full load current of alternator, by adding lamps in lamp board. The results obtained are shown in Fig. 12.

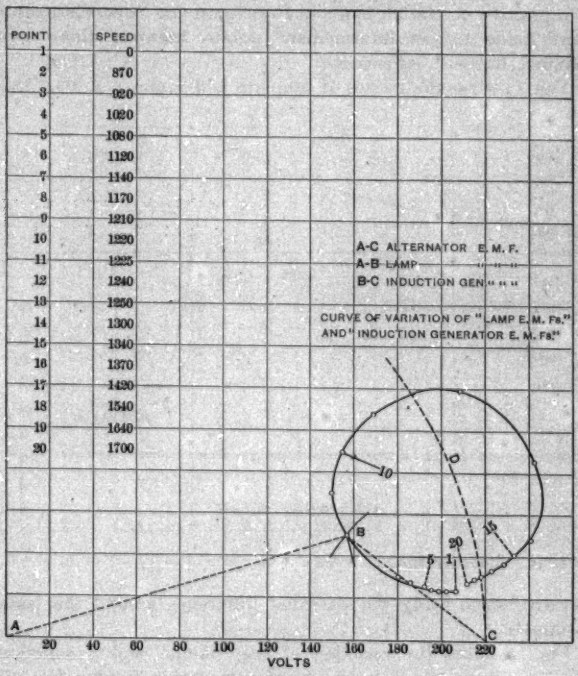


FIG. 15.

In the second test the speed of the induction generator was varied between wide limits, the alternator E. M. F. being maintained constant, and also the number of lamps connected in series with the two machines.

The results are shown in Figs. 13, 14 and 15.

In Fig. 13, the results show that the potential difference around the induction generator terminals increased from 28 volts at rest, to 125 volts at a speed of 1,235 revolutions per minute, and that a slight increase in speed caused the machine to supply E. M. F., in other words, to act as a generator. This change, it will be noticed, is very sudden, going from a negative maximum to a positive maximum in about 10 revolutions. Several attempts were made to get intermediate points between these two maxima, but without success.

Fig. 14 gives the curves of amperes and "lamp E. M. F's.," this

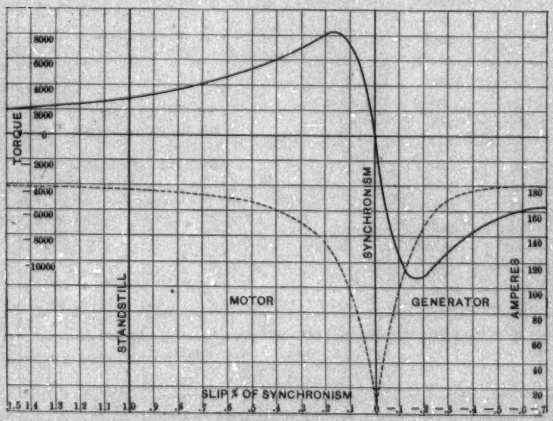


FIG. 16.—Induction Motor. Speed Curves. $Z = .1 - 3j$. $Y = .01 + .1j$.

last expression being the potential difference around the lamp terminals.

Fig. 15 shows the variation of the "lamp" and "induction generator E. M. F's.," polar co-ordinates being used. For all points within the dotted arc D, the induction generator causes a drop in potential, and for all points without this arc the induction generator supplies potential to the system. The method of construction of this curve is simply the location of the vertices of the different triangles of E. M. F's. In this figure the points of the curve are numbered and the corresponding speed readings given in table.

It is interesting to note the similarity between the curves in Figs. 13 and 14 and the one for torque and slip, given in Fig. 16, taken from Mr. Steinmetz's paper on "Induction Motors." While the results, shown in these curves, were taken under somewhat different conditions, still the action, when the motor is driven near synchronism, is strikingly similar.

A paper to be presented at the 15th General Meeting of the American Institute of Electrical Engineers, Omaha, June 20th, 1898.

[ADVANCE COPY—SUBJECT TO REVISION.]

THE EVOLUTION OF THE LINE SIGNAL.

BY ARTHUR VAUGHN ABBOTT.

A careful observer at the first great American Exposition, the Philadelphia Centennial, might have noticed a fine wire running along a gallery in Machinery Hall, terminating at either end of the building in a small closet. Continued investigation would have disclosed a curious trumpet shaped apparatus into which the wire extended, whereby an attentive listener could, with the aid of a vivid imagination, distinguish articulated sentences delivered at the other extremity of the wire. Such was the first public exhibition of the telephone, whereby the transmission of speech by electricity became a fact, and in the quarter of a century which has since elapsed, electrical arachnids have been so industriously spinning a copper web that upwards of 600,000 miles of telephone wire now cover this continent, placing some 400,000 persons in talking relations with each other, and annually transmitting about 900,000,000 messages. But the invention of the telephone was only half the solution of the transmission problem, for the instrument itself was useless without the necessary connecting circuit. Receivers and transmitters are now much the same that they were twenty years ago, but a vast amount of skill and ingenuity has been expended upon apparatus for rapidly, effectually and economically placing subscribers in talking relations with each other and the telephonist of to-day is essentially a "Transmission Engineer."

When telephonic communication is limited to two persons, it is simplicity itself to connect them by the requisite circuit, and provide proper signals to attract attention to the reception of messages, but when the number of correspondents in a single

locality is to be numbered by thousands and tens of thousands, it is physicially and financially impossible to erect a line from each station to all of the others, so early telephone undertakings developed the idea of the "central office," to which lines from all subscribers in the immediate vicinity converge, equipped with the necessary paraphernalia for interconnecting at pleasure any pair of circuits. To carry out the exchange system, the first requisite is to provide each circuit with some form of apparatus whereby the person desiring to send a message may gain the attention of the central office attendant. As the early telephonists

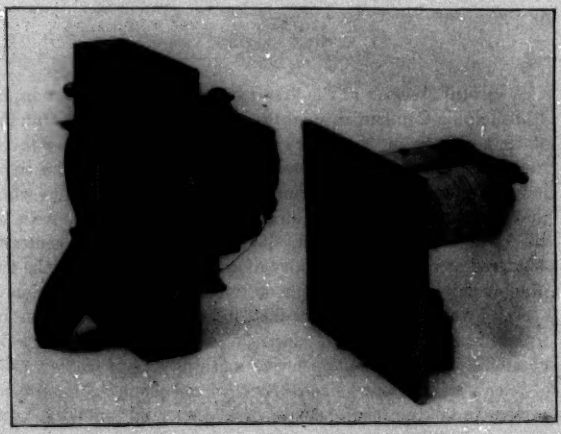


Fig. 1.

comprised many recruits from the "locksmith and bell-hanger school," to which the degree of "electrician" had recently been added on account of a supposed familiarity with the installation of electric bells, it is not surprising that the earliest line signal was the then crude form of hotel annunciator. It consisted, as is shown on the left of Fig. 1, of a pair of bulky spools wound with cotton covered wire, over which an armature was pivoted, carrying a catch that normally retained in a vertical position a door, or shutter. Electrical excitement of the spools released the catch and allowed the shutter to fall forward, disclosing a name

or number painted upon its inner surface. From the falling shutter, this piece of apparatus obtained the euphonious name of "drop," and as originally designed it was exceedingly clumsy, occupying a space of 2″ x 3¼″, so but a very slight exhibition of mechanical skill sufficed to refine and compact it to a notable degree, developing the form shown upon the right-hand of Fig. 1. When it was attempted, however, to set annunciators of this description in sufficient proximity to each other to place before a single operator such a number of lines as it was possible for one person to adequately serve, much difficulty was experienced, due to the cross-talk developed by magnetic leakage between the spools of adjacent signals.

FIG. 2.

Increased mechanical skill and electrical knowledge obviated the cross-talk difficulty by ironcladding as shown in Fig. 2, and in this form, known as the "tubular drop," the line signal remained *in statu quo* for a number of years, and is still very widely employed. But in spite of improved mechanical and electrical details the tubular drop proved itself to be anything but an efficient signal. The falling shutter frequently did not attract the notice of the operator, even though the latter were careful and attentive. Accumulation of dust, either around the trunnions supporting the armature, or under the shutter hook, prevented the shutter from falling, and if adjusted for maximum sensitiveness, the slightest earth current, or the discharge from a summer thunder-shower was often sufficient to display the signals

of an entire switchboard. It is necessary to manually restore the shutter every time it falls, thus necessitating a considerable expenditure of time and energy for this purpose, and so the next step was the so-called self-restoring drop shown in Fig. 3.

Here great skill has been displayed in mechanically designing and constructing the signal. The head of the drop is provided with a double shutter, one thick and heavy of iron, retained in its place by a lever catch connected to the rear armature; the other, a thin outer flap of aluminium that normally conceals the number painted upon the iron disk. Electrically, as shown in Fig. 4, there are two spools. The right-hand winding is designed

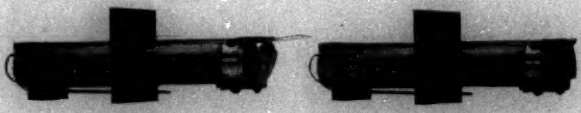


FIG. 3.

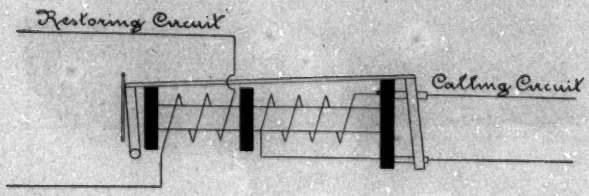


FIG. 4.

for the calling circuit, and when excited attracts the rear armature of the drop, lifting the catch and allowing the heavy iron shutter to fall forward, thus raising the aluminium flap and disclosing the number. The other winding comprises the restoring circuit consisting of a smaller spool which may be excited by a battery controlled by the operator in front of whom the drop is placed, the magnetism thus developed being sufficient to pull the heavy iron shutter back into its place, allowing the aluminium flap and catch to return, thus restoring the drop to its calling position.

Line signals should possess five characteristics.

1st. Certainty in operation.

2nd. Ability to immediately attract attention.

3rd. Positiveness in the information conveyed.
4th. Compactness.
5th. Automatic Operation.

The attempt to simultaneously attain all of these features has given rise to hosts of devices which, for purposes of classification, may be roughly divided into three groups:

1st. Electric Signals.
2nd. Electro-Mechanical signals.
3rd. Luminous Signals.

The self-restoring drop is the best illustration of a signal of the first class. The excitation of the tripping coil by the subscribers'

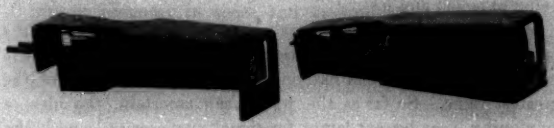


FIG. 5.

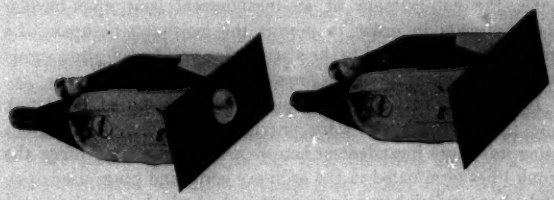


FIG. 6.

ringing current exhibits the signal, while with the second circuit the operator electrically restores the signal to its normal position. The second division is exemplified by the devices shown in Figs. 5 and 6. The contrivance illustrated in Fig. 5 is a particularly successful one, as it is reliable, compact and automatic. It consists of a spool surrounded by a pivoted sheet iron armature. Normally, the armature occupies the position shown at the right-hand of the illustration and is out of sight, but on exciting the spool the magnetization induced causes the armature to rise into the position shown at the left of the illustration, displaying the number painted upon the face of the armature. From the method of operation, this signal has received the suggestive cognomen of

"tip-up." Unfortunately this signal is only well adapted for use where battery currents are employed, as it is not so effective with alternating currents, and in order for the signal to remain visible the magnet must be continuously excited. The arrangement shown in Fig. 6 is ingenious and seems to possess much merit, but has never received a wide application. The usual exciting spool is supplied with a pair of projecting pole-pieces which surround a slightly inclined tube. In this tube a silvered steel ball is placed and the tube so inclined that normally the ball rolls out of view. When the spool is excited the magnetization developed causes the ball to roll forward and to project through a plate covering the orifice of the tube, which also prevents it from being expelled and lost.

Many forms of luminous signals have been suggested but the only one in successful operation at present is the miniature incandescent lamp which has, on the whole, proven itself of the greatest value. So far as the writer can ascertain, the idea of employing incandescent lamps for signal purposes arose with Mr. J. J. O'Connell of the Chicago Telephone Co., who suggested in 1888, the employment of an incandescent lamp upon burglar alarm circuits in order to permit the legitimate occupant of a protected room to send to the alarm office an identification signal whereby his entrance to the premises could be made known.

In the latter part of the Summer of 1890 some of the trunk lines in use in Chicago were provided with disconnect signals using miniature incandescent lamps. At the receiving office the trunk lines terminated in the ordinary cords and plugs used upon a telephone switch-board, in front of each of which an incandescent lamp was located. So long as the line was in use a relay in conjunction therewith was excited by battery current flowing over the line, and its armature drawn up. As soon as conversation was completed the operator at the originating office withdrew the plug from the trunk line jack at that office, thus opening the battery circuit and causing the relay to be de-energized; the release of the relay armature completed a local battery circuit, illuminating the incandescent lamp. The operator at the receiving office noticing the illumination of the lamp was thereby informed that conversation was completed and instructed to remove the trunk line plug. As the local lamp circuit was carried through the operator's listening key, the placing of this key in the testing position extinguished the lamp. A very short experience served

to demonstrate the superiority of incandescent lamps over any other form of signal, for the illuminated lamp was at once so positive and distinct as to instantly attract the attention of the operator, thus markedly accelerating switch-board work. As a result of this experience, most of the trunk lines in the main office of the Chicago Telephone Company were rapidly supplied with lamp signals.

In the spring of 1893 it was proposed to re-equip a large multiple board in the Chicago exchange with self-restoring drops, but the expense and difficulty of this change caused the management to hesitate, and at this time the writer suggested the employment of incandescent lamps as subscribers' line signals on the ground of greater economy in space, and in installation, as well as improved signal efficiency. More conservative telephonists doubted the wisdom of such a recommendation, fearing that the numerous relay contacts would be a perpetual source of trouble, and no further steps were taken with this project. In the winter of 1894 a new type of switchboard known as the "express board" was introduced. As this board was designed to meet the wants of metallic circuit subscribers, particularly those whose telephonic business was large, it was decided to provide incandescent lamp line signals. The circuit of this board was so arranged that on the removal of the subscriber's receiver from its suspending hook, the movement of the hook closed the circuit and allowed battery current to flow through the line. This current actuated a relay at the exchange, in the local of which the signal was placed, situated behind a ground glass shade upon which the number of the subscriber was painted. The illumination of this disk displayed a very distinct and conspicuous signal that immediately attracted the operator's attention. Notwithstanding some apprehension as to the possibility of trouble with relay contacts the service given by the lamp signal was, on the whole, so satisfactory as immediately to concentrate telephonic attention upon the use of lamps, and within the four years that have since elapsed, the superiority of the incandescent lamp for this purpose has been so fully demonstrated that nearly all of the large switchboards which are at present being constructed are completely supplied with signals of this description.

As the service to which the incandescent lamp when used as a signal is subjected is peculiar, the employment of lamps upon telephone switchboards has afforded lamp manufacturers a new

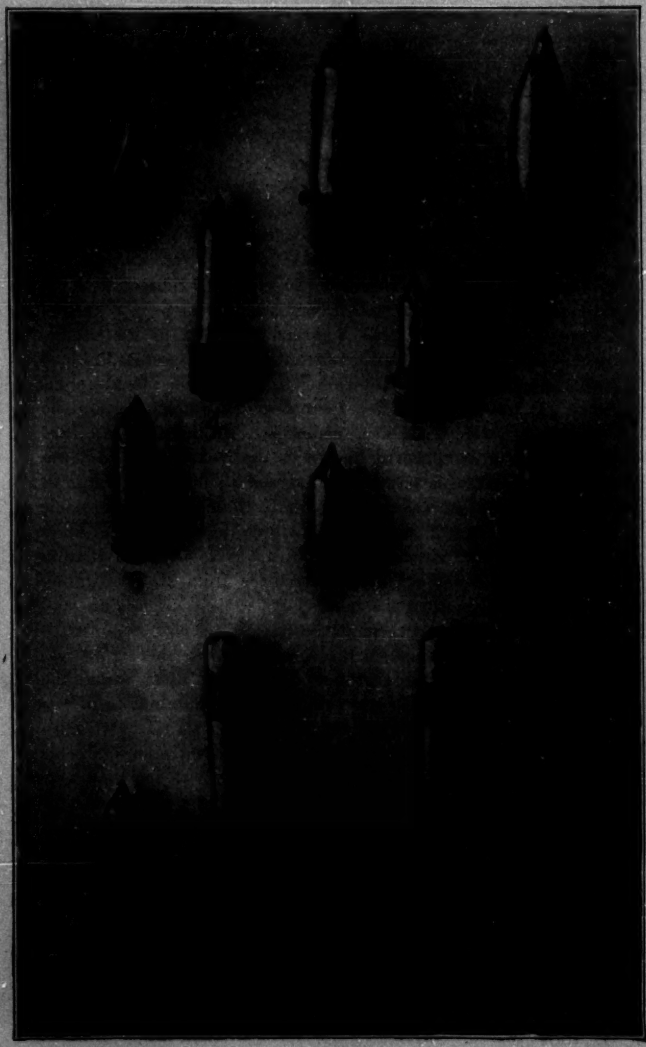


FIG. 7.

problem. In the early experiments ordinary miniature lamps of low voltage were employed. As these lamps were bulky and consumed a large amount of current, the first attempts at improvement were directed to obtaining a smaller and more economical lamp. The various changes through which the lamp has passed are illustrated in Fig. 7, in which No. 1 is the original type of lamp employed, Nos. 11 and 12 are those which experience has now demonstrated to be the most successful. Lamp No. 1 had a bulb about one inch in diameter and was about two inches long. Electrically it was designed for a four-volt circuit requiring over one-half ampere, and giving about two C. P. Lamps Nos. 11 and 12 are $1\frac{1}{4}$ inches long with a bulb $\frac{1}{4}$ inch in diameter. They are designed for a 24-volt circuit, consume about .1 ampere and yield over $\frac{1}{4}$ C. P. The early lamps were designed entirely

FIG. 8.

to fit a socket supplied with a screw thread, as exemplified in Fig. 8.

The socket consisted of a brass tube, in the bottom of which a screw base was placed matching the thread upon the lamp. A cap carrying a ground glass shade covered the front end of the tube, while from the rear two conductors extended, one fastened to the thread of the socket, thus connecting with one of the lamp leading-in wires, while the other was arranged to make contact with the other leading-in wire in the center of the lamp base. This conductor was supplied with a spiral spring, the pressure of which it was expected would lock the lamp in its place and constantly maintain good electrical contact. This expectation was not fulfilled, for one of the greatest troubles encountered has been the loosening of the lamp in its socket, due to the almost inappreciable jarring to which all buildings are subjected. Lamps

in sockets of this description will gradually unscrew, opening the electrical circuit, they are then incapacitated as signals and considerable complaint will arise. Lamp sockets of this kind were designed for lamps Nos. 4 to 7, inclusive (Fig. 7), and could be placed on wooden strips for mounting in the switchboard on one inch centers. The various changes in form through which the lamp bulb has passed are illustrated in Fig. 7, showing the gradual steps whereby the size of the bulb has been reduced until, as shown in Nos. 11 and 12, the present lamp occupies no greater volume than that required by a spring-jack and may be inserted into sockets composed of pairs of springs arranged on a rubber strip. Such sockets so closely resemble standard spring-jacks that they are called "lamp-jacks," for they occupy the same space, and may be secured in the switchboard in the same manner. By placing alternate strips of lamp sockets and jacks, the subscriber's line signal and answering jack may be more closely associated than has previously been possible, space economized in the switchboard and service accelerated.

At first it was considered desirable by lamp manufacturers to preserve a large volume in the bulb of the lamp in order to secure an adequate vacuum. To attain the desired compactness, the bulbs of the lamps were made smaller and longer, as is shown in Fig. 7, numbers 2 and 7 inclusive. Gradually, improvements in manufacture were sufficient to reduce the bulb of the lamp to a smaller diameter than that of the screw base, and it at once became apparent that some other method of securing the lamp in its socket would still further contribute to compactness. The next step was the construction of an exceedingly small bulb lamp, shown in Fig. 7, No. 13, which was secured to a wooden base carrying two metal rings to which the leading-in wires of the lamp were soldered, the complete lamp being shown in Fig. 7 at 8. Whether the theory that the small sized bulb militates against a sufficient vacuum is correct, or not, it has been impossible to definitely ascertain, but experience has shown that lamps having bulbs as small as is indicated in No. 13 were not so successful as large ones. In order to retain the compactness thereby attained, a long and slim bulb was made, as shown in Nos. 9 and 10. In this lamp an additional advantage was secured by placing the filament at the end of the glass opposite the seal. This end could be blown hemispherically and quite thin, thus allowing a maximum of light to be transmitted. Lamps of this description

were chiefly made for four-volt circuits, the filament consisting of a very thin thread of carbon stretched between two platinum wires. These leading-in wires were fused through the sides of the bulb and soldered to two bits of thin copper forming the contact strips that were cemented to the glass tube. Some difficulty was experienced in the peeling off of these pieces, and, in addition, the amount of platinum in the leading-in wires was so great as to considerably increase the cost of such lamps. So, from motives of economy, a partial reversion toward earlier forms has taken place in the present standard lamp, as shown in Fig. 7, numbers 11 and 12. The bulb is a glass tube about $\frac{1}{4}''$ in diameter and $1\frac{1}{8}''$ long. For four volt lamps a straight filament is used, but for all others a horse-shoe shape. Only sufficient platinum is employed in the leading-in wire to pass through the glass, and in other respects the general details of manufacture correspond closely to the best practice in standard lamp construction. The base of the lamp is a bit of boxwood, so shaped as to readily enter the springs forming the socket, and arranged to prevent the lamp from turning as it is pushed into place. The earliest lamps were all designed for low voltage circuits, rarely more than four volts. Presently it became apparent that lamps of longer life, together with an economy in conducting circuits could easily be attained by the use of higher voltages, and the next advance was secured by making lamps for 10 and 20 volts, while at present 12 and 24-volt lamps are standard.

When first used as signals, the lamps were placed in the local of a relay actuated by current passing over the line to which the signal was attached. In the early attempts the ordinary telegraph relay was employed armed with extra heavy platinum contacts to prevent burning. It seemed possible to obviate the expense of installing and maintaining this relay by an appropriate lamp directly in series with the subscriber's line, so placed that the removal of the telephone from its hook should close the circuit and light the lamp. This experiment was first tried in the Chicago express office and, on the whole, worked passably well, there being a comparatively small mortality of lamps due to short-circuits and other accidents. A continuation of this experiment was attempted in another office but resulted disastrously as the lamps were burned out so rapidly as to cause an excessive maintenance expense. The occasion of this failure is to be ascribed to a preponderance of aerial lines in the last case.

A wind-cross occurring near the office on a long line would subject the lamp to an abnormal voltage and either injure or immediately destroy it. In an express office where the lines were chiefly underground, little or no trouble was experienced from this cause, and in an office in St. Louis, similarly equipped with lamp line signals placed directly on the subscriber's line, the use of a very heavy filament lamp obviated the difficulty experienced with injury to lamps. But while economy in installation due to the omission of the relay is eminently desirable, present experience decidedly indicates that the relay is essential to the successful lamp signal.

FIG. 9.

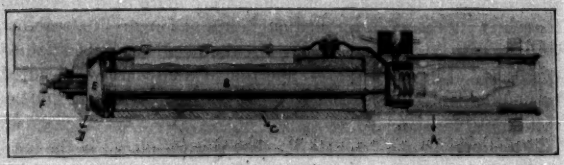


FIG. 10.

For this purpose rather an interesting type has been evolved in an attempt to consolidate the relay, lamp and socket into a single piece of apparatus and at once obtain a device which should be absolutely dust proof and free from trunnion trouble. A general view of this relay which was developed largely with the assistance of Mr. M. A. Edson of the Chicago Telephone Company, is shown in Fig. 9 and a section in Fig. 10.

From the latter illustration it will be seen that the front end of the "lamp relay," as it is termed, consisted of a brass tube A, forming the lamp socket, carrying the screw at the bottom, supplied with round glass cap, carrying the subscriber's number. From the rear of the socket extends a core B, of soft iron sur-

sounded by an iron tube C, between which the winding space was obtained. A hemisperical brass cap D covered the end of the iron tube, retained in its place by an appropriate set screw. The inside of this cap was hollowed out, containing a piece of soft iron E, in the form of a frustrum of a cone, that constituted the armature. As this piece of apparatus was set horizontally the armature rested upon its edge in an unstable position, tending to settle backwards and rest against the back of the cap. The excitation of the windings magnetizes the core and attracting the armature, draws it up sharply. A thin platinum spring placed on the front face of the armature serves to make contact with a corresponding platinum point set in the center of the core. As the core is insulated from everything but one lamp wire, while the other is connected to the cap, the excitation of the relay lights the lamp. By this device a relay was obtained, the moving parts of which were completely dust-proof. All trunnions or hinges were obviated and the armature merely resting upon a sharp edge secures a minimum amount of friction. Notwithstanding the compactness and completeness of this piece of apparatus, it has been found desirable to disassociate the relays and lamps, placing the relays in a case by themselves, away from the switchboard.

Probably the most gratifying feature in the evolution of the line signal has been the improvement in the life of miniature lamps. When the luminous signal was first proposed the question of cost of lamp renewals was at once raised. As each lamp was only illuminated for a few seconds at a time the service seemed equivalent to constant flashing, and ominous prophecies were plenty as to the short life to be expected. It was confidently asserted that such small lamps would either immediately burn out, or else so rapidly become dim as to be valueless as signals. Some tests were immediately instituted in the hope of eliciting information on these points. There was no question but that the candle power of the lamps gradually declined, and photometric measurements showed that when a lamp failed to emit at least .05 candle power, it became of no value as a signal.

To determine the resistance to flashing, a number of lamps were connected in circuit with a clock pendulum and flashed at intervals one second for nearly a month, with no signs of serious injury. During this test the lamps were illuminated more than a million times, so it seemed certain that a reasonably long life

could be expected. Experience has since shown this conclusion to be justified, for in the main office switchboard in Chicago, there are now lamps at work that have been in service for upwards of three years and seem still to be in fairly good condition. Observation has further shown that accidental injuries rather than old age, is the cause of most lamp failures.

Much delay was experienced in obtaining the first lot of switchboard lamps so on reception they were immediately placed in service without inspection. The rapid failure of a large number of these caused no small degree of consternation and it was de-

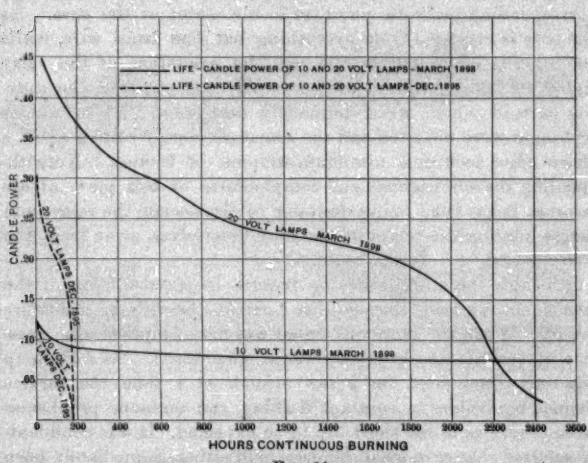


Fig 11.

termined to initiate a systematic inspection. As a result, all lamps are now examined: 1.—Mechanically, to see if they will fit the sockets. 2.—Electrically, to determine if they have the proper voltage and consume current within the specified limits. 3.—Optically, to ascertain if they emit the required candle power, and absorb the proper watts per candle. Also a certain number of lamps from each lot are placed on life test and burned to destruction.

In Fig. 11 four curves are given that show at a glance the improvement that has taken place in lamp life. The dotted lines

are curves of life and candle power of ten samples, each of 10 and 20-volt lamps tested in 1895, while the full lines are the results from twenty samples of 10 and 20-volt lamps of the present date. In 1895, only 200 hours' life could be expected, while from the latest tests the 20-volt lamps show 2400 hours' life before failure, and the 10-volt lamps show 2500 hours, and are still burning with indications of considerable additional life.

In Fig. 12 is an interesting comparison of the rate of failure of sample lamps tested in 1897, and at the present time. Curves 1, 2, 3 and 5 are for 20-volt lamps of different makers, while curve

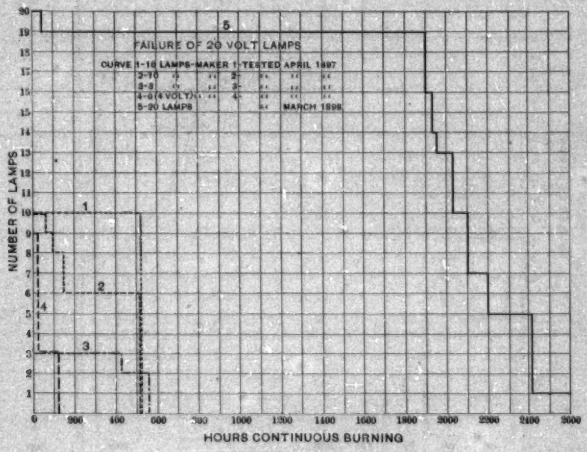


FIG. 12.

4 is for four-volt lamps. The ordinates are the number of lamps on test at the various times represented by the abscissæ in hours, so that the life of each lamp is plainly shown. The four-volt lamps failed almost immediately, only yielding upwards of 50 hours. The 20-volt lamps, curves 1, 2 and 3, tested in 1897, gave upwards of 600 hours life as an average, those indicated by curve 1 showing the greatest uniformity. In curve 5, the results of ten 20-volt lamps tested this spring are plotted; one lamp failed after 40 hours, but the filament was manifestly defective at the start. Nineteen lamps lasted over 1900 hours and one lamp over 2500 hours.

In Fig. 13 the curves of 10-volt lamps tested in a similar manner are shown. These results indicate a much greater irregularity for the 10-volt lamps. The lamps of curve 3 make a bad showing, only one lamp lasting over 40 hours; those of curves 1 and 2 are much better, indicating a life of 400 hours, but still are much below the life shown by the 20-volt lamps. Curve 4 represents the results of tests upon twenty 10-volt lamps recently manufactured. This curve is a straight line for 2500 hours, show-

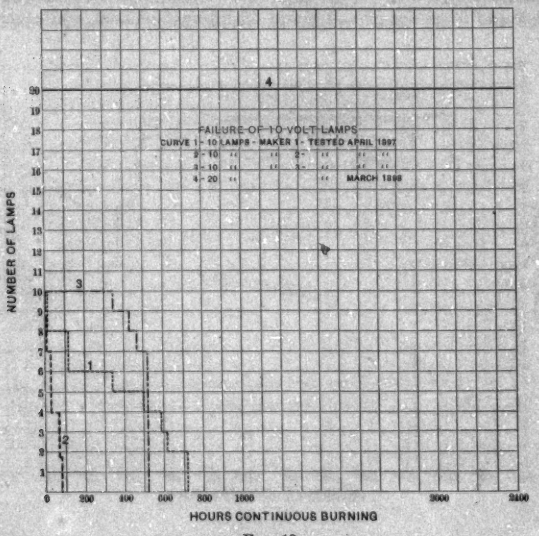


FIG. 13.

ing that so far no failures have occurred, and from the appearance of the filament, it is reasonable to expect several hundred hours more life.

In Fig. 14 some information is plotted relative to candle-power, life and voltage of 20-volt lamps. Curves 1, 2 and 3 show the relation between the life and candle-power of 20-volt lamps when placed upon circuits of respectively 21, 20 and 18½ volts. The left-hand scale in candle-power applies to these lines. Curve No. 4 is a life-voltage curve, for which the scale of volts on the

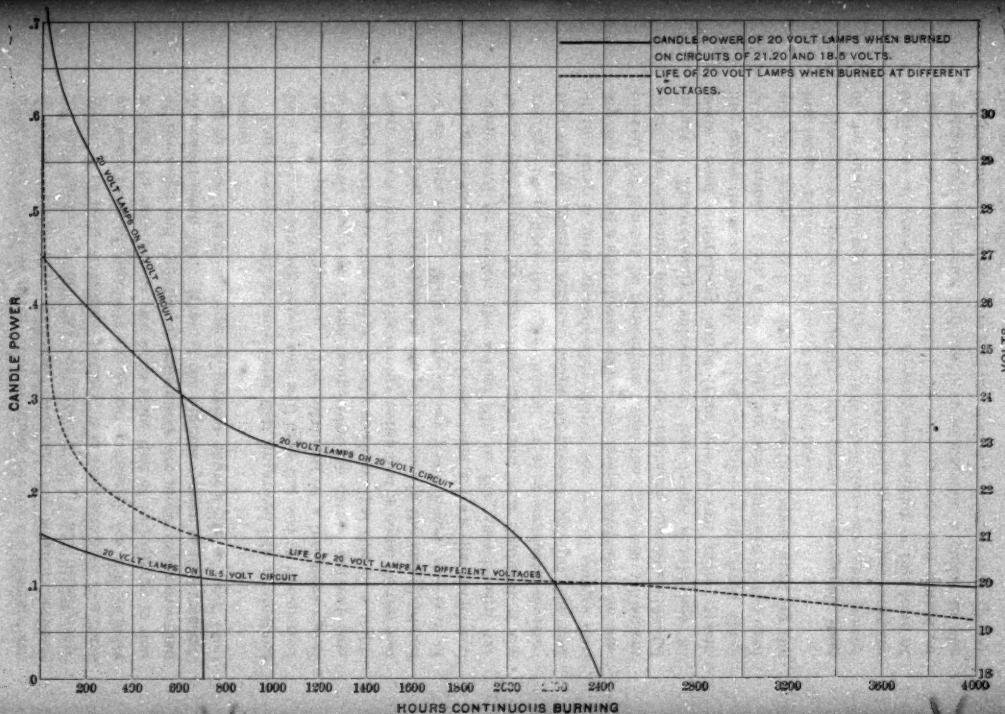

CANDLE POWER OF 20 VOLT LAMPS WHEN BURNED ON CIRCUITS OF 21.20 AND 18.5 VOLTS.
LIFE OF 20 VOLT LAMPS WHEN BURNED AT DIFFERENT VOLTAGES.
20 VOLT LAMPS ON 21 VOLT CIRCUIT
20 VOLT LAMPS ON 20 VOLT CIRCUIT
LIFE OF 20 VOLT LAMPS AT DIFFERENT VOLTAGES
20 VOLT LAMPS ON 18.5 VOLT CIRCUIT
CANDLE POWER
VOLTS
HOURS CONTINUOUS BURNING
200 400 600 800 1000 1200 1400 1600 1800 2000 2200 2400 2800 3200 3600 4000
0 1 2 3 4 5 6 7
18 19 20 21 22 23 24 25 26 27 28 29 30

FIG 14.

right-hand should be used. For all these curves the abscissæ are in hours. These lines strikingly illustrate the relation between life and candle power. As switchboard lamps are usually supplied by 10 cells of storage battery, it is impracticable to always maintain them at precisely a constant potential, for the pressure is likely to vary one or two volts from that of full charge to that of discharge and regulating resistances are more expensive than decreased lamp life. Even at the highest pressure, it is interesting to note that the lamps at present show a life of over 1400 hours.

Lamp experts will at once infer that improved life is obtained by a lamp of low efficiency and will urge the cost of the current as an insuperable objection. The total cost of a luminous signal is evidently the cost of the lamp, plus the cost of current consumed during its life, and it is easy to show that this is a minimum when the cost of the energy consumed is equal to the cost of the lamp. Switchboard lamps now cost in the neighborhood of 45 cents each. With a life of say 2,000 hours, the cost of energy is about 60 cents, but it has been shown that accidental causes tend to reduce the actual life in the switchboard far below that shown by laboratory tests and it is doubtful if the average lamp lives for 1,200 hours. Unless accidental destruction can be greatly reduced below the present amount, the cost of the lamp is now considerably above the cost of energy during its life and a still lower efficiency could profitably be employed, for it must be remembered that the thick heavy filament in the low efficiency lamps is the best insurance against accidental injury. Theoretically, subscribers' line lamps should last 25 years; cord supervisory lamps from one to two years; trunk line lamps the same length of time, and pilot lamps from three to six months. Such a life as this has been obtained in the cord and pilot lamps, but it is doubtful if the theoretical limit for the line lamps will ever be closely approximated.

Even in the present probably only partially developed state of the luminous signal, some 20,000 subscribers are now served upon boards thus equipped, and switchboards for as many more are in process of construction. Already the adoption of this form of signal has materially modified the arrangement and design of switchboard apparatus, and has been so efficient a factor in the improvement of telephonic service that it is difficult to prophesy the end of its effect.

In conclusion, the writer is only too glad of an occasion to express his appreciation of the promptness with which the various lamp manufacturers have taken up the problem of the line lamp signal, in the solution of which they have spared neither pains nor expense.

A Paper to be presented at the 15th General Meeting of the American Institute of Electrical Engineers, Omaha, June 30, 1898.

[ADVANCE COPY.—SUBJECT TO REVISION.]

A MODERN ELECTRIC CENTRAL STATION.

BY GEORGE A. DAMON.

The plant of the Imperial Electric Light, Heat and Power Company, of St. Louis, Missouri, is interesting at this time, as it is a good example of the present tendency in electric power station practice.

The designers of this installation were not burdened at the outset with the complication of an old system of distribution, nor were they confronted with the necessity of adapting their new station to the use of the generating machinery already on hand. Everything was to be modern, and the engineers in charge of the work were free to adopt the plans which seemed best suited to the existing conditions. The territory to be supplied with electrical energy was a down-town district, and the character of the load to be expected included arc and incandescent lighting as required in stores, hotels, office buildings, and theatres, with more than the ordinary amount of motor work for elevators and light manufacturing establishments.

In planning an installation for a commercial enterprise of this character two conditions are imposed upon the designer. First, the service supplied the customer must be absolutely reliable. The earning capacity of the whole enterprise depends primarily upon the ability to furnish a continuous electric service, and any impairment or interruption would be a serious drawback to the success of the undertaking. After reliability has been secured, the second problem is to deliver to the customer electrical energy at the least possible cost, considering interest upon first investment, maintenance, and all other factors which enter into operating expenses. The engineering success of the enterprise, viewed

from a commercial standpoint, will be measured in the proportion that the completed work meets the two requirements of maximum reliability and economy, consistent with minimum investment.

The system which has been adopted as the one best fulfilling all the conditions in this particular case, involves the use of the direct current 220-volt incandescent lamp; the location of the station as near as possible to the center of distribution; the operation of generators capable of delivering current at a potential of 500 volts, a special arrangement of the generating units, and the installation of a storage battery in the plant to act as an accumulator, and also in parallel with the generators as an equalizer. The generators are run at a potential enough greater than 440 volts to overcome the line loss, and the motor circuits are at present operated at this potential. The storage battery across the outside mains allows 220-volt incandescent lighting feeders to be distributed over the district.

It is not the purpose of this paper to discuss the 220-volt lamp question, any more than to indicate that the adoption of this lamp, after a careful investigation, for a central station of this size and importance, shows considerable faith in its possibilities. The greatly reduced investment required for copper distributing feeders for a 220-volt system from that required by the usual direct current 110-volt system, the enlargement therefore of the territory which could be supplied, as well as the better quality of light and the possibility of operating 220-volt motors on the same circuits and thus making it possible to connect in many motors all ready in use in the city, were the main advantages which led to the adoption of the 220-volt lamp. The comparative lower efficiency and higher first cost of the lamp as compared with the 110-volt type was recognized, but these points are yielding to development, and it is thought that the difference between the two lamps in respect to efficiency and price is sure to become much less as the demand for the new lamp increases.

Local conditions made the location of the plant a much simpler problem than is usually the case. To have secured free water for condensation purposes would have removed the plant much too far from the center of distribution. The difference between drayage charges on the coal delivered to the station on its present site, and the cost of bridge tolls and switching

charges which would have been necessary if the coal had been delivered directly from the cars, is only 15 cents per ton in favor of the latter plan, and this is not enough to justify the extra expense in feeders which would have been required to secure the location of the plant on a railroad switch. No doubt, therefore, was entertained in regard to the advisability, in this particular case, of carrying the fuel of the plant in wagons from the railroad to the station at the center of supply, rather than transmitting the product of the plant over copper conductors the same distance.

The advantages of supplying all classes of service from one type of generator, thus simplifing the distribution circuits as well as the arrangement of the station, and at the same time increasing the economy of the plant by reducing the amount of both fuel and labor required were not forgotten. At periods of light loads it is possible to furnish the entire output for incandescent lamps, arc lights and motors from one generator, while during the hours of minimum demand the load may be carried by the accumulator. The recent rapid progress of the storage battery in this country has been brought about only after a discussion which has made the economic advantages of a battery auxiliary in connection with a central station system, matters of common information, and therefore they need not be discussed at great length at this time.

The battery in this case is to be used in the ordinary manner. During periods of light loads it will accumulate a charge to be given out again at times of sudden demand, or in order to carry the plant over the peak of the load. The ability of a battery to respond immediately to unexpected demands, is one of the most advantageous features of an accumulator auxiliary. The necessity of carrying a sudden increase in the load arises continually in central station operation, and usually this condition requires that a reserve of boilers, engines and generators be held always in readiness under steam. Many central station managers keep a generating unit turning slowly over, ready to take its share of the load at a moment's notice, thus greatly increasing what may be called the "stand-by" losses of the plant, and sacrificing the economy of operation of the station to the integrity of its service. An accumulator of proper proportions will take care of these extra unexpected loads with much better fuel economy and at considerable saving in labor and worry.

The battery acts in the same manner in carrying the peak of the load. This peak usually extends over a period of from one to three hours. and in an ordinary plant the equipment which is necessary to supply the extra demand at this time, is inoperative during the remainder of the day. The interest on the investment, the fixed charges and the allowance for depreciation this part of the plant are the same, whether the machinery is running for two hours or for the entire 24 hours of each day. The interest factor is thus greatly increased in the cost of producing the amount of energy required to carry the extra demand at the peak of the load, over the cost of the remaining average output of the station. Add to this the fact that the plant is necessarily working at poor efficiency on account of a constantly varying load of short duration, and the effect of the peak of the load upon the earning capacity of the station may be appreciated. The storage battery auxiliary improves these unfavorable conditions, first, by evening up the load line, and thus increasing the efficiency of the generating plant, and second, by decreasing the total amount of investment required. The advantages of convenience and reliability of an accumulator in a central station cannot be questioned. Whether or not it will improve the efficiency or decrease the total amount of required investment in any particular case is influenced by local conditions, principally the character of the load line. In the plant being considered the expected peak of the load extends over a period of little more than one hour, which is a condition especially favorable to the use of a storage battery, though the "factor of safety" which the battery adds to the plant is an advantage of nearly sufficient importance to justify the investment.

Coming to the generating station proper, it is found that every precaution known to modern practice has been adopted to deliver at the switchboard as much as possible of the energy originally contained in the fuel. A central station is at its best a very wasteful institution, delivering to the distributing circuits, under its best working conditions, but from five to seven per cent. of the energy originally supplied to it from the fuel Where the evident possibilities of the increasing efficiency are so great, the study of the economics of a modern electric plant is an imporant one.

Fig. 1 shows a cross-section of the station and indicates the general compact arrangement of the plant equipment, which was made necessary on account of the cost of real estate in the vi-

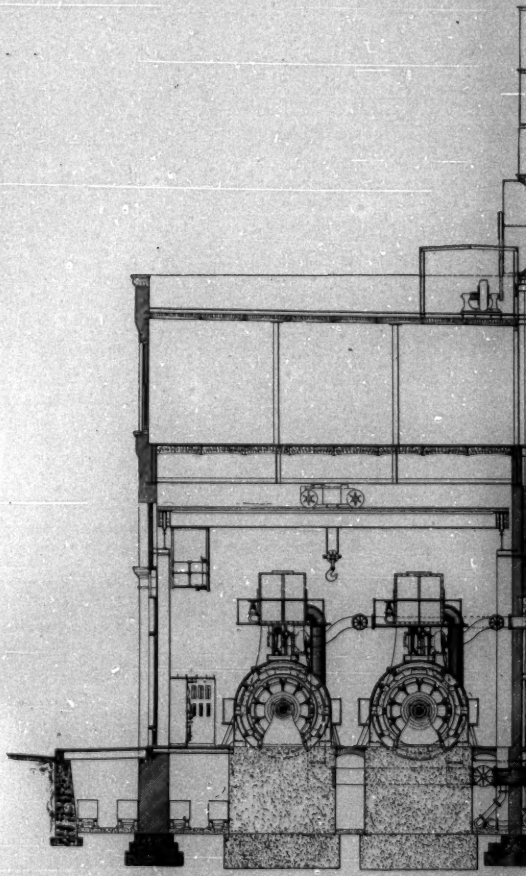
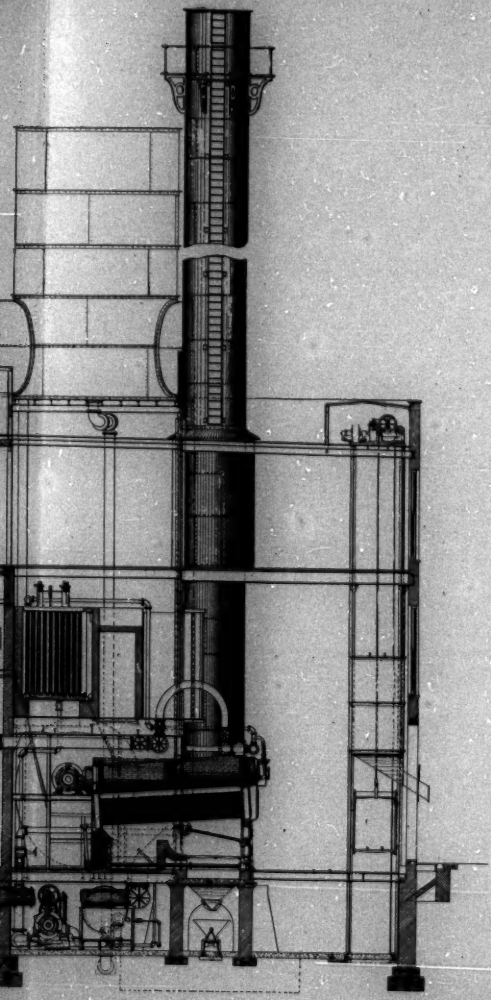


Fig. 1.—Cross-sect



ction of Plant.

cinity of the site. The engine-room and the boiler rooms are upon the ground level, which arrangement ensures plenty of light, good ventilation, and convenience in operation. The engine room basement is used for the storage batteries, while in the boiler-room basement is located the coal-storage bin, the ash-handling apparatus, the piping, pumps, hot-well and condenser. A second floor is added to the building for the offices of the company, for the storage of supplies and for a machine and repair shop.

The furnaces of the boiler are of the down-draft type, thus securing the advantage of skilful hand firing, and at the same time avoiding the cooling of the gases, by the frequent opening of the furnace door. The water-cooled grates furnish some additional heating surface to the boiler and the general arrangement ensures a better combustion of fuel than is ordinarily obtained. This results in reducing the amount of ash and preventing the production of smoke, which is an absolutely essential feature with a city installation. Considerations of availability and cost, limit the fuel supply to coal from Southern Illinois mines, which produce a bituminous slack containing about 11,000 B. T. U. per pound. As the wagons deliver the coal directly on the boiler-room floor, automatic coal-handling machinery was not thought to be a good investment.

Horizontal water tube boilers were adopted on account of their quick steaming properties, the comparatively small amount of floor space occupied by large capacities, their economy of fuel, and their safety under high steam pressures, the pressure adopted for this plant being 175 pounds. In determining the size of the boilers, due allowance was made for the fact that they were to be operated in conjunction with economizers, and their capacity was reduced accordingly. The question of economizers was thoroughly considered, and they were adopted only after careful estimates had been made, which demonstrated that the probable saving of fuel effected by their use would be sufficient to cover the usual allowance for depreciation, insurance, etc., and yet leave a very good margin for interest on the investment. As no steam-driven auxiliaries are to be used in the plant, there will consequently be no other feed-water heater besides the economizer. The economizer for each battery of two 375 H. P. boilers consists of 320 pipes, and the heavy weight is carried on special iron framework which places the economizer at the back and above the boilers as shown in the figure. The flue passages, and stack con-

nections, are so arranged and fitted with dampers that the flue gases can pass either directly to the stack, through the economizer to the stack, or through the mechanical draft fan in series.

A mechanical draft fan is now considered by many engineers, a necessary adjunct to a power station, and especially is this true if economizers are used, as in this case the flue gases escaping from the boilers can be robbed of nearly all their latent heat without impairing the draft. To the fact that a saving is thus effected in the usual large waste due to a chimney, the mechanical draft fan has the further advantage of furnishing a ready means of controlling the draft according to the load upon the plant, so that the boilers may be worked with comparative economy at light loads, or may be forced far beyond their rated capacity without too seriously affecting their economy. Unfortunately in this particular plant, the city ordinances required a stack which would carry the gases up above the surrounding buildings. For this purpose a steel chimney has been built, but it is a disadvantage rather than a benefit as far as the question of draft itself is concerned.

The engines are compound-condensing, and are the only steam using parts of the entire station equipment, all auxiliary apparatus being operated by means of electric motors. The steam thus passes directly from the boilers into the engines with a minimum amount of steam piping exposed for radiation losses. The steam from the engines exhausts into a surface condenser placed in the basement below the boiler-room, from which it is pumped into the hot-well and through the economizers back into the boilers.

The condensing water which flows through the tubes of the condenser is taken from the bottom of a cooling tower located upon the roof at an elevation of 50 feet above the condensing apparatus. After this water passes through the condenser it is forced back to the top of the cooling tower by means of centrifugal circulating pumps. The water circuit is arranged so that the weight of the descending column is used to balance the weight of the ascending column of water. The work actually required of the circulating pump, therefore, is represented by the amount of power required to raise the water through the height of the tower, or about 30 feet, and the net result is the same as if the tower had been located on the ground level.

The plans for the completed station contemplate a total capacity of 5,000 K. W. There are at present, however, installed in the station

but two engines, one of 750 H. P. and the other of 1,500 H. P. capacity. These engines are of the cross-compound marine type of massive design and run at a speed of 150 R. P. M. They are expected to develop an indicated horse power hour on less than 14 pounds of water. Each engine is designed, however, to stand a continuous overload of 100 per cent. This will be developed by means of admitting high pressure steam directly into the low pressure cylinder. The conditions under which it is expected to operate the engines in this manner will be referred to later. The cylinders are placed side by side, each piston acting upon a separate crank. Between the cylinders is located the receiver which is provided with copper reheating coils. The high-pressure cylinder is steam-jacketed on the barrel, and both cylinders have a steam-jacket on the heads. Both receivers and cylinders are protected with a non-conducting covering over which there is a steel jacket held in place by bands. Each engine is provided with a heavy fly wheel located in the center of the frame between the cylinders. The throttle-valve is of the flanged lever type and is operated from the starting platform. The speed is regulated by a shaft governor operating an eccentric connected to the cut-off valves of the high-pressure cylinder, and in addition to this governor, each engine is provided with an additional automatic safety-valve, designed for the protection of the engine in case the speed should be abnormally increased for any reason. The engines also have a speeding device attachment by means of which they may be brought to the same rate of speed under frictional load as under full load.

The engines are directly connected to three 500 K. W. generators and two 50 K. W. boosters. These dynamos are of the multipolar type with ironclad bar wound armatures. Provision is made for sliding the field casting parallel with the shaft a sufficient distance to allow the field coils to be removed and the armatures reached for repairs. The brush holders are carried in a circular iron frame, arranged to be moved by means of a threaded rod and a hand wheel, so that the position of the brushes may be adjusted. An additional hand-wheel is provided for lifting the brushes from the commutator.

The engines and generators are connected by means of the "Arnold System" This system of power station design is well known, but its application to this particular station will be of interest. The general idea of the system is to improve upon the usual direct-connected unit principle by mounting each gener-

tor in such a manner that it can be operated by more than one engine. Fig. 2 shows partly in cross-section and partly in elevation, the line of engines and generators as they will eventually appear as the output of the station is increased. Three engines are shown, the capacity of the center engine being 1,500 H. P. and double that of either end engine. Each of the four large generators is rated at 500 K. W. while each of the small central dynamos, shown located between the larger machines, is a low-voltage booster of K. W. capacity. The central engine therefore has a capacity sufficient to operate two of the generators when running at its most economical load, while either end engine is proportioned to supply but one generator with power under normal working conditions. The use of the boosters in this plant is incidental, and occurs at periods of light loads only, so that no additional engine capacity over that demanded by the main generators is required for their operation. Under all ordinary running conditions the units of this plant are manipulated as usual, an effort being made to vary with the load the number of engines running, so as to keep those in operation at any one time working as nearly as possible at their rated capacity. In this respect the operation of the plant follows general practice. In a station containing independently-direct-connected units, however, if any piece of the generating machinery gets out of order, it disables the entire unit and thus the commonly accepted principle which underlies the selection of the size and number of units for a direct-connected station is to have in reserve and always ready to be put into immediate service, engine and generator capacity sufficient to take the place of the largest single unit in operation. Thus, with a station containing five generating units of equal size, the station capacity which may be safely relied upon is the combined output of but four of the units, the fifth unit being either held in reserve or undergoing repairs. An additional investment of 25 per cent. in station equipment over that actually required by the demands of the service is necessary, therefore, in this case, in order to ensure reliability. Where less than five independent units are required, the percentage of the investment which may be charged up to the account of reliability is increased. Thus, with three units, the reserve is seen to require an investment of 50 per cent. over that actually needed by the demands of service. It has already been pointed out that one of the greatest drawbacks to central station work is the amount of equipment which must be provided in order to take care of the

Fig. 2.—Arnold System.

Fig. 3.—Detail of Arnold System.

occasional maximum demand. When to this burden is added an additional investment of from 25 to 50 per cent. in order to secure reliability, it is easily seen that the earning capacity of the station may be seriously affected. One of the advantages of this system of power-station construction is that the extra reserve unit is not required. Each generator is not rigidly connected to its corresponding engine but is in such a manner that it may be operated by more than one prime mover. Thus in Fig. 2, the cen-

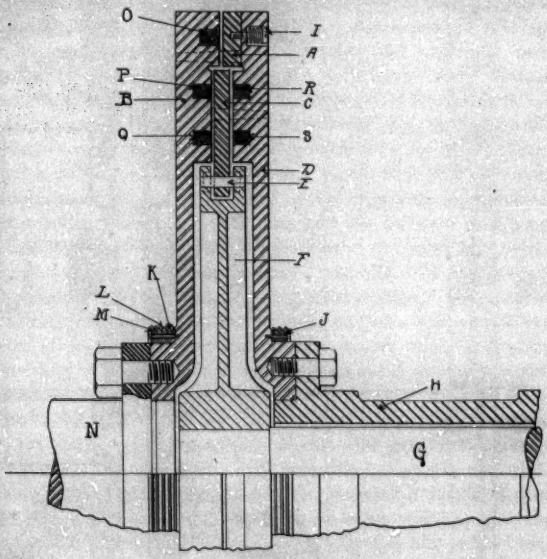


FIG. 4.—Detail of Magnetic Clutch.

tral engine may run either one or all four of the generators as well as the two boosters, while the smaller end engines can reach either or both of two generators and also one booster.

The method of mounting and connecting the dynamos which makes this flexible arrangement possible is shown more clearly by Fig. 3. Each generator armature is mounted on a hollow shaft or quill Q, and is carried independently in the bearings P P. The booster armature is attached to the solid shaft E. This shaft is made of forged steel and extends through the hollow genera-

tor quills but without touching them, so that it is free to revolve in the bearings D. It is by means of this shaft that any engine can reach the generators not contiguous to it, but under all normal operating conditions, this shaft does not turn so that this system of connecting involves no more friction and requires no more lubrication than the ordinary arrangement. The coupling on the end of the shaft permits of three combinations: The engine may be directly connected with the quill of the adjacent generator allowing the interior shaft to remain idle, or the solid shaft may be attached to either the engine flange or to the generator quill. The latter two combinations allow the transfer of power peculiar to this system.

The coupling or clutch is of particular interest as it is said to be the first magnetic clutch applied upon a large scale to power station work. Fig. 4 shows a cross-section detail of this clutch. One part, B, is bolted to the engine flange, N. A corresponding part, D, is bolted to the generator quill, H, and this part carries an annular armature, A. Radial arms F, keyed to the solid shaft O, carry an interior armature C. Each coupling is provided with five circular coils of magnet wire O, P, R and S. These coils are electrically connected to contact rings J, K and L, carried on insulating bushings bolted to the hubs of the clutch. Stationary brushes which are connected through a system of switches to the storage battery 'bus bars make it possible to send a current around any coil. The parts of the clutch containing the coils are made of cast-steel of high permeability so that when a current is sent around any coil, the surrounding metal becomes a magnet and the armature facing the coil is immediately magnetically attracted. The two surfaces of the clutch parts being held in contact by a pressure of at least 80 pounds per square inch, it becomes possible to transmit power from one to the other. Thus to connect engine shaft N to generator quill H, a current is sent through coil O in that part of the clutch B attached to the engine shaft. Armature A which is attached by means of strong studs to that part of the class B which is securely bolted to the generator quill, is immediately attracted and over-coming the small resistance of the spring G, the two parts become firmly attached to each other, making it possible to run the generator from the engine. If it is desired to revolve the interior shaft by means of the engine, the current is sent through coils P and Q in parallel, attracting armature C, while if the solid shaft is being turned by the other engine and it is desired to connect the shaft to the

generator quill, then the coils R and S are energized and the armature C is attracted in the other direction. The circuits to these coils are all controlled by means of switches on the main switchboard. These switches are so arranged that when the circuit of any coil is broken, the discharge of the extra current is taken up through a carbon resistance, and the insulation of the coil is therefore protected from the liabitity to puncture. When the current in any coil is discontinued, spiral springs have been provided to overcome the residual magnetism and bring the armature back to its original position.

If both parts of the clutch which are to be attached are stationary, the connection becomes an easy matter, but if one part is revolving, it becomes necessary to put the other part in motion before the current is sent through the coils. This is accomplished by starting the dynamos as motors and bringing them up to speed before closing the magnet coil switch. For this purpose, regular starting-boxes have been installed, one for the boosters and the other for the generators, so that any machine may be thrown into or out of service directly from the switchboard. The result of the whole arrangement is, that a number of combinations between the engines and the generators becomes possible. Under all normal running conditions, the center engine will run its two adjacent generators and the outside engines will operate the end generators. Each booster may be run when necessary, from either of two engines. The entire line of machinery may be connected together, so that all the engines and generators operate in unison as one large unit, the speeding attachment on the engines making it possible to operate them satisfactorily in this manner.

In case of accident to any generator that particular machine may be stopped without stopping its corresponding engine, and the electrical load will be immediately taken up by the other generators, but without overloading the engines. In case of accident to any engine, the ability of the other engines to carry an overload of 100 per cent. may be taken advantage of, if necessary, without overloading any of the generators. Thus the center engine is capable of carrying all four of the generators fully loaded while the smaller end engines can operate two of the generators, if necessary, in case of a break-down of the larger engine. The plant may, therefore, operate at period of maximum load, at its full rated capacity, depending entirely upon the advantages of the system of arrangement for a reserve in case of a breakdown to any part of the equipment.

The flexibility and reliability of the system may be compared to that of a belted plan and this has been secured without the use of an unsightly arrangement of belts or the complicated system of levers and pivots necessarily connected with mechanical clutches. The advantages of the belted plant have thus been combined with the advantages of economy and compactness of the direct-connected plan and many of the disadvantages of both arrangements have been eliminated.

All the variable speed motors operating the auxiliary equipment of the plant are designed to run at two voltages, the maximum being 440 volts, and the other voltage being one-half the maximum. Further speed regulation is obtained by changing the field excitation by means of a small resistance in the field circuits. The use of armature resistance is thus entirely obviated. A switchboard containing the switches and controlling apparatus for the pumps and fan motors has been placed conveniently in the boiler-room and another auxiliary board has been placed in the engine-room to control the circulating pump and cooling-tower motors as well as the circuit leading to the crane which spans the engine-room. The use of economically controlled electric motors for running the pumps, fans and other auxiliaries of the plant is expected to result in a considerable saving of fuel.

The storage battery is located in the basement and has a capacity of 2,000 ampere-hours at the normal rate of discharge, but is capable of standing a discharge at the rate of 500 K. W. for one hour. The station, therefore, as it now stands, can carry a maximum of 2,000 K. W., three quarters of which can be taken care of by the generators, and the remaining quarter being supplied by the battery. The battery consists of 280 cells, 100 of which are end or regulating cells, by means of which the potential may be regulated, the voltage depending, of course, upon the number of cells in circuit.

Fig. 5 shows a diagram of the electrical connections of the station. The generators are compound-wound, but are so designed that they will give a potential of 500 volts with shunt excitation only, as it is desirable when operating with storage batteries to run the generators as shunt dynamos. As extra switch has been added therefore to each generator panel which is used to cut out the series winding and connect the equalizer lead directly to to the positive 'bus bar. Referring to the diagram: when switches 6 and 8 are closed, and 7 and 9 are open, the generator

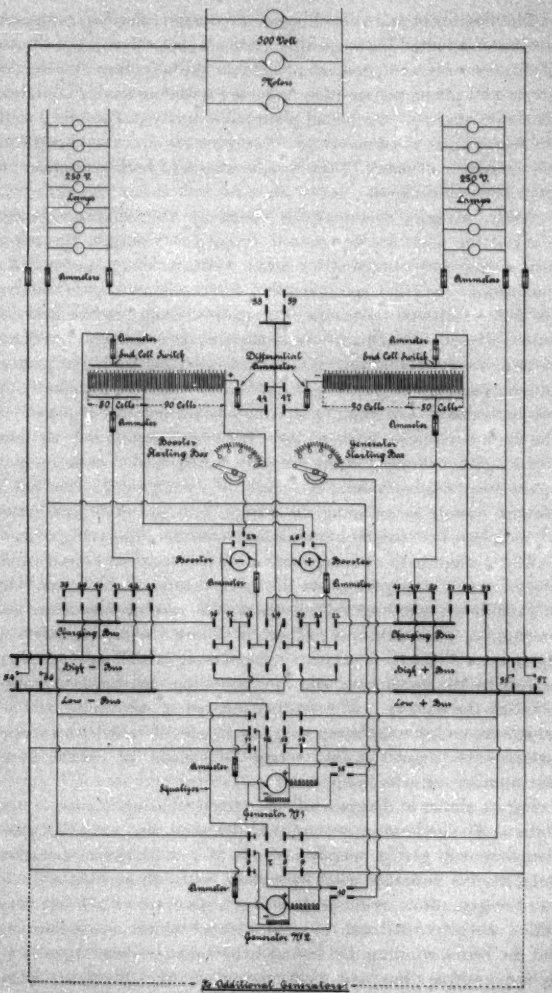

500 Volt
Mains
250 V.
Lamps
250 V.
Lamps
Ammeter
Ammeter
Ammeter
Ammeter
End Cell Switch
Differential Ammeter
Ammeter
End Cell Switch
50 Cells
90 Cells
50 Cells
50 Cells
Ammeter
Ammeter
Booster Starting Box
Generator Starting Box
Booster
Booster
Ammeter
Ammeter
Charging Bus
Charging Bus
Light − Bus
Light + Bus
Low − Bus
Low + Bus
Ammeter
Equalizer
Generator No 1
Ammeter
Generator No 2
To Additional Generators

FIG. 5.

is running as a straight shunt machine, while with 6 and 9 closed, but with 8 open, the dynamo is connected to the omnibus bars as a compound generator, and may be thrown in multiple with the other dynamos of the plant by closing equalizing switch 7.

When starting up a dynamo, the shunt fields are excited directly from the omnibus bars through a special field switch. when the generator is taken out of service, this field switch is automatically opened, leaving the field coils connected as in a shunt machine, and thus avoiding the discharge of the coils.

The switchboard is a double potential board so that the feeders extending to the outlying districts of the territory may be supplied with current at a higher potential than the shorter feeders. All switches on the generator and feeder panels are arranged so that when thrown up they connect with the high potential omnibus bars, and when thrown down, they connect with the low potential bars. All positive switches are placed on the right of the panels.

The battery is connected in two independent parts and is provided at each of the two outside ends with two end-cell switches. These end-cell switches may be connected through suitable knife switches to either the "high" or to the "low" omnibus bars or to a charging bar. Combinations of switches numbered on the diagram (Fig. 5) from 16 to 22, allow the boosters to be connected between either bar and the charging bar. The boosters in the plant are used to charge the batteries when it is desired to give the batteries a heavier charge than they would ordinarily receive when working in parallel across the mains at the regular voltage. Without in any way interfering with the performance of the battery as an equalizer across the system, the boosters can be placed in series with the batteries and thus any amount of current can be forced through the cells. The condition for this operation occurs at periods of light demand upon the station, which is the time the battery can be most economically charged. The voltage of the boosters ranges from 0 to 130 volts. The two boosters may be placed in series and can thus be substituted for either leg of the battery, allowing the replaced half of the battery to either accumulate an extra charge in case the two sides of the system have become so badly unbalanced as to have discharged one side of the battery excessively, or that part of the battery may be entirely cut out for repairs. In fact, the boosters

may take the place of the entire battery if necessary, acting as an equalizer across one leg of the system, giving or taking current, as occasion requires.

The switch-board consists of eleven panels of black enamelled, polished slate. The three generator panels are each provided with the usual complement of switches, a rheostat, a cutting-in galvanometer, a circuit-breaker, an ammeter, and a voltmeter. One booster panel and two battery panels are necessary for the manipulation of the battery auxiliary. The battery panel is provided with hand-control wheels which connect to the end-cell switches so that the potential of the battery can be charged from the front of the board. It is also possible to operate each end-cell switch by means of a small motor directly attached to the switch spindle. This motor is controlled by a push button on the front of the battery board, and an indicator shows how many cells are in service at any time. Low-reading voltmeters and voltmeter switches allow the potential of the end cells to be ascertained conveniently from the front of the board. A voltmeter panel contains four station wattmeters which register the total output of the plant. Three feeder panels are at present employed to distribute the current, and these panels each contain six feeder-switches and six corresponding ammeters. A regulating voltmeter panel is located at the end, and at right angles to the rest of the board in a position where it can be easily seen by the switch-board attendant.

No radical departure is claimed for the design of this station. It has been described simply to show the latest solution worked out by conservative engineers of the problem of the generation and distribution of electrical energy. It is thought to be of interest because it presents a combination of the following widely discussed features of modern central station practice: (1) Improved furnaces; (2) Water-tube boilers; (3) Mechanical draft; (4) Fuel economizers; (5) Electrically-driven auxiliaries; (6) Cooling tower condensing system; (7) Arnold system of power station construction; (8) Battery auxiliary; (9) Two-wire distributing system, and (10) 220-volt lamps.

A paper to be presented at the 15th General Meeting of the American Institute of Electrical Engineers, Omaha, June 30th, 1898.

[ADVANCE COPY—SUBJECT TO REVISION.]

THE GRAPHICAL TREATMENT OF ALTERNATING CURRENTS IN BRANCHING CIRCUITS

With Special Reference to the Case of Variable Frequency.

BY HENRY T. EDDY.

1. Let a simple harmonic E. M. F. whose constant maximum pressure is E volts be applied to the common terminal of the two branches of the split circuit shown in Fig. 1.

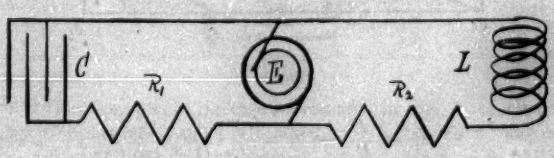


Fig. 1.

Each of the branches, designated as 1 and 2, is taken to have in it either a condenser or a self-inductive resistance in series with an ohmic resistance of R_1, or R_2, ohms; but no mutual induction between the branches is supposed to exist. After treating the special cases in which there is one kind of resistance only in each branch, besides ohmic resistance, viz: either a condenser in one branch, and a self-induction in the other; or a condenser alone in each branch; or self-induction alone in each branch; the general case in which both branches have in series condensers and self-inductive resistances as well as ohmic resistances will be made to depend upon the special cases first treated.

In case the pressure E acts between the terminals of branch 1, having a resistance of R_1 ohms and a capacity of $C = 1 / K$ farads, the maximum current of I_1 amperes may be found from the clock diagram, Fig. 2, in which I_1 is the chord $O\ P$, of the semi-circle whose diameter is $O\ E_1 = E/R_1$, and θ_1 the lead of I_1 is given by the equation.

$$\tan \theta_1 = \frac{K}{R_1\, \omega} \tag{1}$$

in which $\omega = 2\, n\, \pi$ is speed or angular velocity, and n is frequency or number of complete alternations per second.

Now

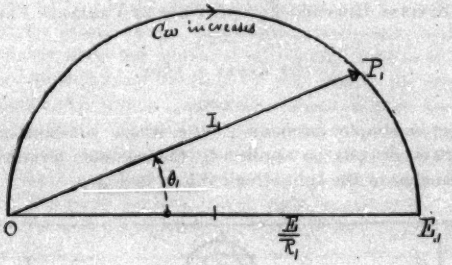


Fig. 2.

$$I_1 = \frac{E}{R_1} \cos \theta_1 = \frac{E}{R_1} \frac{1}{(1 + \tan \theta_1)^{\frac{1}{2}}} \tag{2}$$

Hence I_1 increases from the value zero to E/R_1 as ω and the frequency increases from zero to infinity.

It will be convenient in treating variable frequency to speak of ω as the measure of the frequency.

Similarly, in a second branch having a resistance of R_2 ohms and self-induction of L henrys, the pressure E causes a maximum current of I_2 amperes shown in Fig. 3 as the chord $O\ P_2$ of the semi-circle whose diameter is $O\ E_2 = E/R_2$, and the lag θ_2 is given by the equation

$$\tan \theta_2 = \frac{L\, \omega}{R_2}, \tag{3}$$

But

$$I_2 = \frac{E}{R_2} \cos \theta_2 = \frac{E}{R_2} \frac{1}{(1 + \tan \theta_2)^{\frac{1}{2}}} \qquad (4)$$

hence $I_2 =$ decreases from the value E/R_2 to zero as the frequency increases from zero to infinity.

The semi-circles in Figs. 2 and 3 are the loci of the extremities of the vectors representing the maxima values of the currents I_1 and I_2 in case R_1 and R_2 remain constant, while $\alpha = \tan \theta_1$ and $\beta = \tan \theta_1$ are made to vary from zero to infinity or *vice versa*, either by varying ω alone, or K and L alone, or by varying these all at once.

It is convenient to regard α and β as a kind of coordinates for expressing the properties of split circuits.

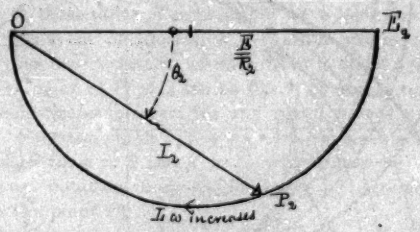


FIG. 3.

2. For example, in the case represented in Fig. 1, of a capacity C in branch 1, and a self-induction L in branch 2, take one coordinate constant, as for instance let $\alpha = \tan \theta_1 = 2$, while $\beta = \tan \theta_2$ is variable. Then if we lay off in Fig. 4 $O\,E_1 = E/R_1$ and $O\,E_2 = E/R_2$ we have $I_1 = O\,P_1$ which vector will be fixed in magnitude and position, so long as α is constant. But P_2, the extremity of the vector $I_2 = O\,P_2$ will describe the semi-circumference $O\,E_2$ from O to E_2 as β decreases from infinity to zero through the successive values shown on Fig. 4.

Hence P, the extremity of the vector $O\,P = I$ which represents the total current produced by E in both branches, or the resultant of I_1 and I_2, describes a semi-circumference whose diameter is equal and parallel to $O\,E_2$, and beginning at the fixed point P_1 the extremity P describes the complete semi-circumference while β decreases from infinity to zero. In like manner are

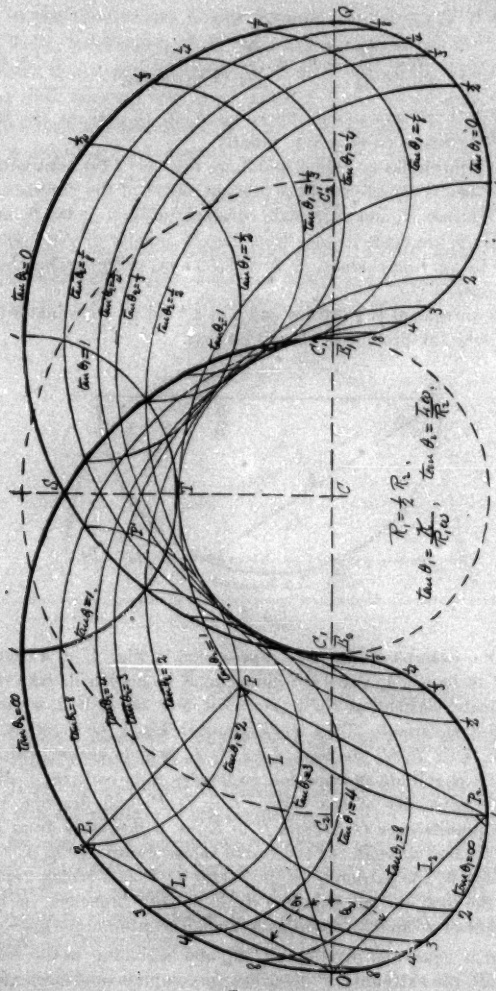


FIG. 4.

to be located the other semi-circumferences along which α has the successive constant values shown in Fig. 4. They are each to be constructed by locating P_1 upon the circumference of the semi-circle $O\ E_1$ in such successive positions that $O\ P_1$ will cause the corresponding co-ordinate $\tan \theta_1 = \alpha$ to assume the required constant values. Their centers all lie on the circumference of the semi-circle $C_2,\ C_2^1$. A set of semi-circles whose diameters are equal and parallel to $O\ E_1$ and whose centers lie on the semi-circle $C_1\ C_1^1$ will likewise begin at successive fixed points P_2 along the semi-circle $O\ E_2$ and will show the locus of the extremity of the vector $I = O\ P$, for successive constant values of $\tan \theta_2$ while $\alpha = \tan \theta$ decreases from infinity to zero, as shown also in Fig. 4. From this point on, we shall for the sake of brevity use the terms circle and semi-circle to designate circumference and semi-circumference.

When both these series of semi-circles have been drawn upon a diagram, for a set of successive numerical values of the co-ordinates lying near together, we then have in effect a table from which the value of $O\ P = I$ can be found by observing at what point the semi-circles intersect for any given values of the co-ordinates. For example in Fig. 4 we have represented the current $I = O\ P$ for $\alpha = 2$ and $\beta = 1$, which may be expressed in abbreviated notation more conveniently by designating this point P as the point (2, 1).

In case the semi-circles of each set are drawn for successive values of the co-ordinates at small numerical intervals, interpolation for intermediate values will be readily effected.

Different diagrams will be required for different ratios of R_1 to R_2; all other changes require a change of scale only in order to measure I.

Fig. 4 has been constructed for $R_1 = \frac{1}{2}\ R_2$. Were $R_1 = 2\ R_2$ while C and L remained unchanged in their respective branches, the diagram would be turned upside down, and be in effect, a reflection of Fig. 4 in $O\ Q$.

Now the two sets of semi-circles are so situated with reference to each other that some of the semi-circles of one set may apparently intersect certain ones of the other set in two points. But if the matter be rightly considered, such double intersections do not really occur; for the area we are considering, is enclosed by the five semi-circles whose centers are at $C_2\ C_1\ C\ C_1^1\ C_2^1$. Hence $S\ E_1$ and $S\ E_2$ are portions of its bounding edges just as

truly as $S\,O$ and $S\,Q$. In fact the area consists of two equal parts or leaves which partially overlap: the first leaf bounded by the three semi-circles whose centers are $C\,C_1\,C_2$, and the second leaf by the three whose centers are $C\,C_1'\,C_2'$. These two leaves are joined along the semi-circle $E_1\,T\,E_2$ which thus forms an *edge of junction* and our tabular diagram would be correctly constructed, were it actually made of two leaves united along this semi-circle and not elsewhere. It will be noticed that every semi-circle of each set touches this semi-circle $E_1\,T\,E_2$ which is therefore the envelope of both sets. Every semi-circle of either set begins at P_1 or P_2 at the edge of the first leaf, and if a point P start at P_1 or P_2 and move along one of these semi-circles, it traverses the first leaf, reaches the point of tangency on the edge of junction, then enters upon the second leaf, and finally reaches the edge of the second leaf. Considered in this way, as it must be, no semi-circle of one set has more than one intersection with any semi-circle of the other set. But any point of the first leaf within the area $S\,E_1\,T\,E_2$ falls upon a point of the second leaf having $O\,P = I$ identically the same for both. In other words, there are two perfectly distinct ways of producing the same total current in case P falls within this area. In case however that P fall within either leaf and outside this area, there is one way only of producing the current $I = O\,P$, while no possible variations of frequency can make P fall outside both leaves.

Consider some point (α_1, β_1) in the first leaf, say (2, 3); then the corresponding double point at the apparent second intersection of these semi-circles is numerically very near $(\frac{8}{10}, 3)$ in the first leaf, and $(2, \frac{1}{3})$ in the second leaf; while the point $(\alpha_2\ \beta_2)$ in the second leaf to which this double point corresponds is very near $(\frac{8}{10}, \frac{1}{3})$. In other words the double point (α_1, β_2), (α_2, β_1) corresponds to both (α_1, β_1) and (α_2, β_2). It also bisects the distance between them, for the center of the semi-circle α_1 $(= 2_1$ say) must be moved the same distance in the same direction to change it to α_2 $(= \frac{8}{10}$ say) as the center of the semi-circle β_1 $(= 3$ say) to change it to β_2 $(= \frac{1}{3}$ say), viz: the distance from one of these points to the double point.

According to this way of viewing the subject, any point, as $(2, \frac{1}{3})$ is regarded as located by starting from O and following successively two of the arcs which form the sides of the curvilinear parallelogram, whose angular points are (∞, ∞) at O, $(\alpha, \frac{1}{3})$,

$(2, \infty)$ and $(2, \frac{1}{2})$ by a geometric addition of sides. It will be noticed that the order of the addition is unimportant, and that the point arrived at, is in the second leaf, whichever order be followed. The actual parallelograms of currents are the chords of sides of the curvilinear parallelogram.

Every possible value of the total current I lies between $O\,Q = E/R_1 + E/R_2$ and zero. There is a minimum value of I for each given value of either of the co-ordinates. For example let $a_1 = \tan\theta_1 = 2$, then the nearest point of this semi-circle to O, may evidently be found by drawing a straight line from O to the center of the semi-circle $a = 2$, and similarly for any semi-circle of either set. The distance from O to the intersection is the required minimum. The corresponding value of the other co-ordinate can be read from the diagram.

For each given value of either co-ordinate there is no maximum value of I, but there is a greatest value at the extremity of each semi-circle farthest from O.

For each given value of either co-ordinate, there is a value of I, giving a minimum lead (or lag), to be found by drawing through O a line tangent to the given semi-circle; but there is no maximum lead (or lag); its greatest value however, is to be found by drawing a straight line from O to that extremity of the given semi-circle which lies nearest to O.

It will be shown later, that for any pair of points situated at the same height above or below $O\,Q$ and at equal distances to the right and left of $S\,T$, the values of their co-ordinates are respectively the numerical reciprocals of each other:—for, example, $(2, 3)$ and $(\frac{1}{3}, \frac{1}{2})$ are so situated.

3. Next consider the case of two branches each of which contains a condenser, or each a self-inductive resistance, in addition to its ohmic resistance.

In Fig. 5 each branch is supposed to contain a condenser, and it is assumed that $R_1 = \frac{3}{2} R_2$. Were both branches to contain self-inductions instead of condensers, the semi-circles would all lie below $O\,Q$. The construction is like that of Fig. 4, except that both θ_1 and θ_2 are on the same side of $O\,Q$.

The semi-circles of both sets are tangent to the semi-circle $O\,T\,Q$ which forms the edge of junction in this case. The other boundaries of the area within which P must fall are the four circles whose centers are $C_1\,C_2\,C_1^1\,C_2^1$, while $S\,O\,T\,Q$ is the portion of the area, where the first leaf which is bounded by three

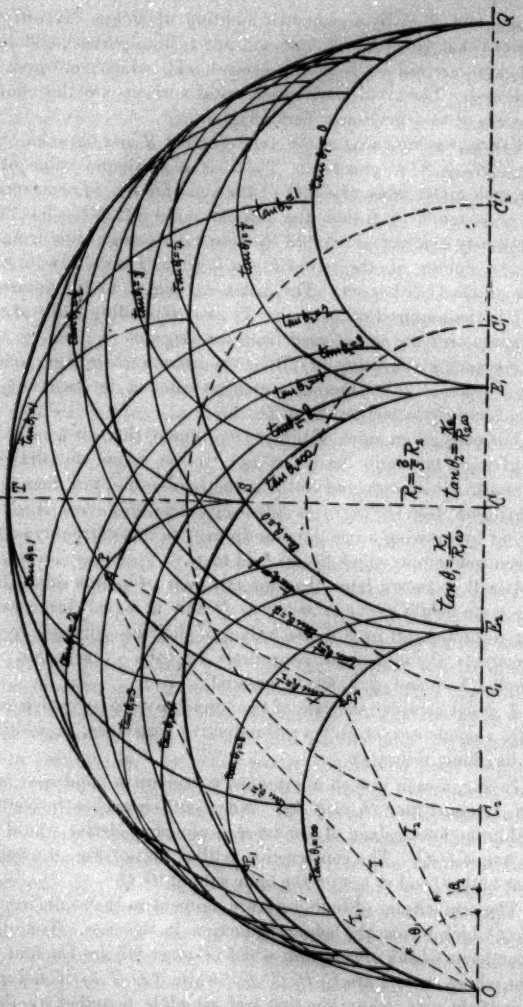


FIG. 5.

semi-circles whose centers are C_2 C C_1^1, overlaps the second leaf, bounded by the three semi-circles whose centers are at C_2^1 C C_1.

Every semi-circle along which $\beta = \tan \theta_2$ has any given constant value, starts in the first leaf from the initial semi-circle whose diameter is O E_2 and ends at the semi-circle in the second leaf whose diameter is Q E_1 as $\alpha = \tan \theta_1$ decreases from infinity to zero; while every semi-circle along which $\alpha = \tan \theta_1$ has any given constant value starts in the second leaf from the initial semi-circle whose diameter is O E_1 and ends in the first leaf at the semi-circle whose diameter is Q E_2 as β decreases from infinity to zero. Since the semi-circles belonging to any pair of given values of α and β, start in different leaves, their point of intersection (α, β) lies on one of the semi-circles further from the starting point than its point of contact with the edge of junction, and on the other semi-circle nearer its starting point. In Fig. 5, the point $P = (2, \frac{1}{2})$ lies in the first leaf, and on $\alpha = 2$ it lies further from the starting point P_1 than the point of contact with the edge of junction, while P lies on $\beta = \frac{1}{2}$ nearer P_2 than its point of contact. The point of the second leaf which coincides with P is approximately the point $(\frac{8}{10}, \frac{7}{2})$. Any double point (α_1, β_2), (α_2, β_1) lies midway between the points (α_1, β_1) and (α_2, β_2), and the point P in Fig. 5 lies midway between the two points whose numerical values are approximately $(\frac{8}{10}, \frac{1}{2})$ in the first leaf and $(2, \frac{7}{2})$ in the second. The proof of this statement is the same as that given in Fig. 4.

There is a maximum value of I for each given value of α or β, and it may evidently be found by drawing a straight line from O through the center of the semi-circle for that given value until it intersects the semi-circle. But there is no minimum value of $I = O\,P$ for each given value of α or β, though there is a smallest value when P is at the initial point of the semi-circle nearest to O.

For each given value of α or β there is a value of I giving a maximum lead (or lag), which is to be found by drawing from O a tangent to the given semi-circle. In Fig. 5, O P is very nearly tangent to $\beta = \frac{1}{2}$. The least value of the lead (or lag) is found by drawing a line from O to the farthest extremity of the given semi-circle, this value is however not a minimum. Any pair of points whose respective co-ordinates are numerical reciprocals of each other lie at equal distances to the right and left of S T and at the same height above (or below) O Q, as will be shown later.

The special case of $R_1 = R_2$ would cause E_1, E_2 and S to fall at C, and so make the two leaves cover precisely the same area. Also in this case the semi-circles $\alpha = \beta$ having equal numerical values coincide, and every point is a double point in which the numerical values of α and β exchange places, *e. g.* (2, 3) and (3, 2) would refer to the same double point. The constructions in Figs. 4 and 5 are specially suited to discuss cases in which the capacity or self-induction of a single branch is made to vary, for then, one or other co-ordinate only varies.

4. Having thus shown the boundaries and contour lines of the two-leaved plane surface within which P, the extremity of $I = O P$, the total current must lie in case of a split circuit, it will be specially useful in case of variable frequency to discuss certain loci obtained by assuming fixed relations between the co-ordinates. These relations arise naturally from the algebraic form of their values as expressed in equations (1) and (3), viz:—By multiplying (1) and (3) together we have

$$\tan \theta_1 \tan \theta_2 = \frac{K L}{R_1 R_2} = G^2 \text{ say}, \tag{5}$$

which is independent of the frequency, *i. e.* the products of co-ordinates for the same value of the frequency is independent of the frequency and is the same for all frequencies.

It is possible to construct the locus for any given value of G, (*i. e.*, when K and L are known) in a convenient manner, as follows: Introduce two auxiliary angles φ_1 and φ_2 such that

$$\tan \theta_1 = G \tan \varphi_1, \text{ and } \tan \theta_2 = G \tan \varphi_2,$$

$$\therefore \text{ by (5), } \tan \varphi_1 \tan \varphi_2 = 1, \tag{6}$$

$$\therefore \varphi_1 + \varphi_2 = 90°. \tag{7}$$

It is evident that the magnitudes in these equations are related to each other in the manner represented in Fig. 6, in which $D A F$ is a right angle, and

$$G = \frac{A B}{O B}, \tag{8}$$

for, $\overline{O B} \tan \theta_1 = \overline{A B} \tan \varphi_1$

and $\overline{O B} \tan \theta_2 = \overline{A B} \tan \varphi_2$.

$A D$ and $A F$ may be drawn at right angles by making them pass through the extremities of the diameter of any circle through A whose center is on $A B$. In this way it is possible to construct

with ease the lead θ_1 and the lag θ_2 of any pair of branch currents I_1 and I_2 that belong together, *i. e.*, that are produced at the same frequency. Should the value of the frequency be required, it can be found from equation (1). In Fig 6 we have taken $G = \frac{1}{2}$. Let us proceed to construct the required locus as shown in Fig. 7, in which lay off $O\,E_1 = E/R_1$, and $O\,E_2 = E/R_2$; and taking any convenient point B, assume A in such a

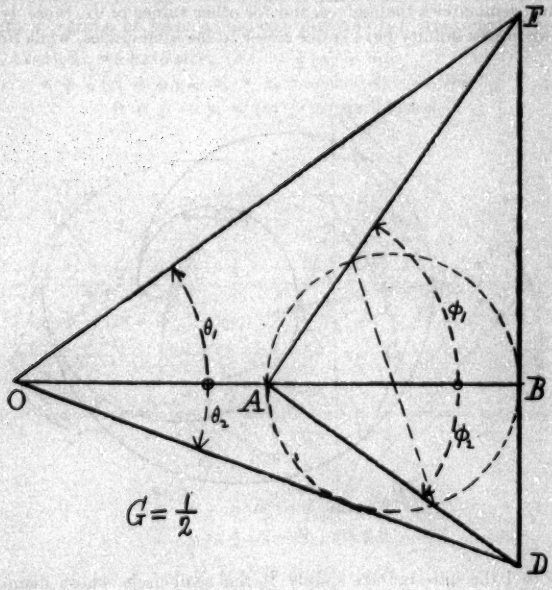


FIG. 6

position as to make the ratio $A\,B/O\,B = G$ assume any particular value desired. In Fig. 7, $G = 2$. Construct θ_1 and θ_2 as was done in Fig. 6, then $O\,P_1 = I$ and $O\,P_2 = I_2$ are the components of the total current $O\,P = I$.

By taking successive positions of lines at right angles to each other through A, the points P_1 and P_2 will traverse the semicircles whose centers are C_1 and C_2, and the point P will traverse

a curve consisting of a single spire reaching from E_1 to E_2. The spire shown in Fig. 7 is the locus corresponding to $G = 2$, in case 3 $R_1 = 4\ R_2$.

The fact that $R_1 > R_2$ causes the upper part of the spire to be the smaller, since the diameter E/R_1 of the semi-circle C_1 is less than the diameter E/R_2 of the semi-circle C_2, and each spire approximates to some extent in its shape to that of the two semi-circles. In fact, when G is infinite the curve consists of these two semi-circles themselves, and for other values of G those lying nearer infinity have spires closer to the semi-circles, while for

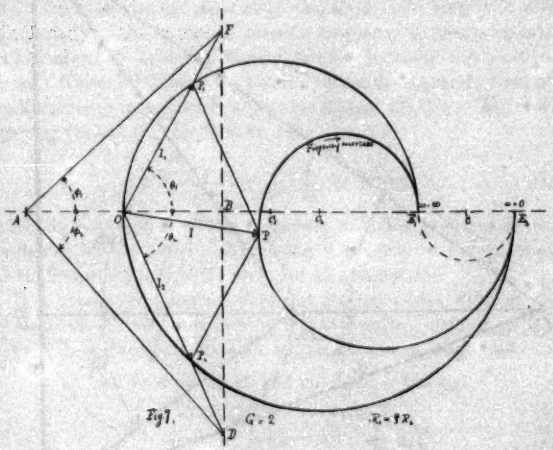


FIG. 7.

$G = 1$ the spire reduces simply to the semi-circle whose diameter is $E_1\ E_2$. Spires for various intermediate values of G are shown in Fig. 9 on the left.

The truth of these statements may be shown as follows:

In case G is infinite, either K or L must also be infinite by equation (5). But, K infinite makes $I_1 = 0$ by eqs. (1); and 2, in which case I coincides with I_2, and the semi-circle on $O\ E_2$ is the locus. While L infinite makes $I_2 = 0$ by eqs. (3) and (4), and the semi-circle on $O\ E_1$ then becomes the locus. In case $G = 1$, the locus is the semi-circle on $E_1\ E_2$, for resolving I_1 and

I_2 into their working and wattless currents, and noting that in this case $\theta_1 + \theta_2 = 90°$ by eq. (5), we have on taking the origin of rectangular co-ordinates at O, and putting $E/R_1 = a_1$, and $E/R_2 = a_2$,

$$I_1 \cos\theta_1 = x_1 = a_1 \cos^2\theta_1,$$
$$I_2 \cos\theta_2 = x_2 = a_2 \cos^2\theta_2 = a_2 \sin^2\theta_1,$$
$$I_1 \sin\theta_1 = y_1 = a_1 \sin\theta_1 \cos\theta_1,$$
$$I_2 \sin\theta_2 = y_2 = a_2 \sin\theta_2 \cos\theta_2 = a_2 \sin\theta_1 \cos\theta_1.$$

But if x and y be taken as the co-ordinates of P reckoned from C as origin, we have since $O\,C = \frac{1}{2}(a_1 + a_2)$

$$\left.\begin{aligned} x + \tfrac{1}{2}(a_1 + a_2) = x_1 + x_2 &= a_1 \cos^2\theta_1 + a_2 \sin^2\theta_1 \\ y = y_1 - y_2 &= (a_1 - a_2) \sin\theta_1 \cos\theta_1 \end{aligned}\right\} \quad (9)$$

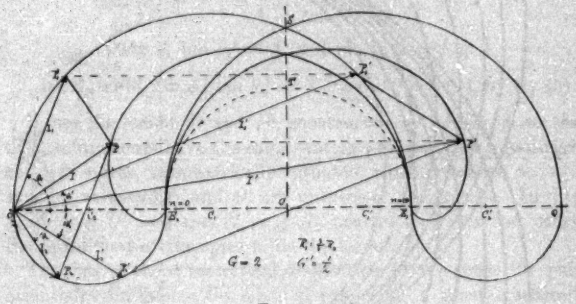


FIG. 8.

$$\therefore x = \tfrac{1}{2}(a_1 - a_2) \cos 2\theta_1$$
$$y = \tfrac{1}{2}(a_1 - a_2) \sin 2\theta_1$$
$$\therefore x^2 + y^2 = \tfrac{1}{4}(a_1 - a_2)^2 \quad (10)$$

and the locus of P in this case is therefore the semi-circle on $E_1 E_2$.

Before proceeding to construct the locus of P for cases in which $G < 1$, it will be first proven that reciprocal values of G give pairs of loci which are equal right and left spires such that they are simple reflections of each other in $C\,S$ of Figs. 8 and 9. For, in Fig. 8, let P_1 and P_1^1 be such that I_1 and I_1^1 have equal wattless currents, as also I_2 I_2^1; then the total currents I and I^1 must have equal wattless currents. Hence P and P^1 are equidistant from E_1 E_2.

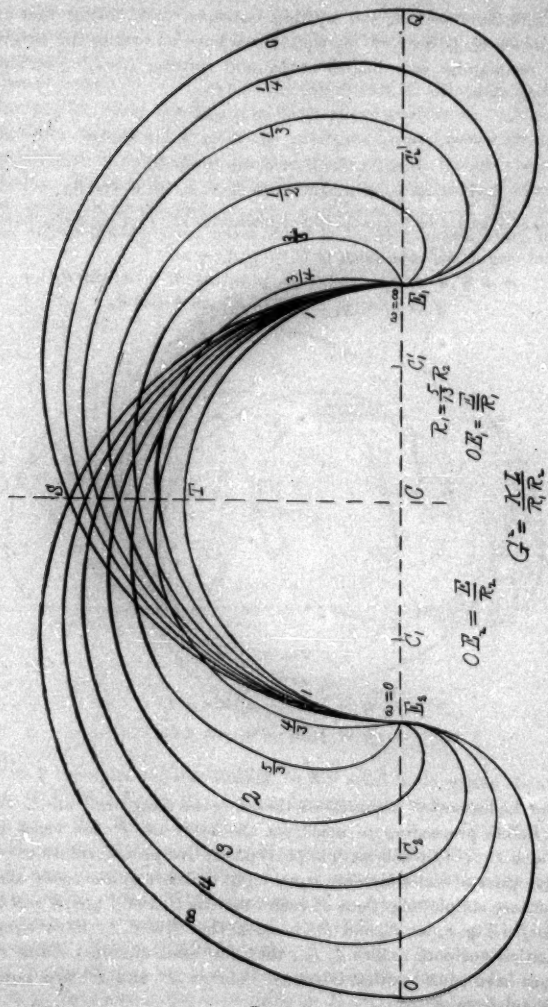

Q
0
1/4
1/3
1/2
2/3
3/4
1
ω=∞
E₁
C₁'
R₁=√(R₂/R₁)
OE₁=E/R₁
S
T
C
OE₂=E/R₂
G=K/R₁R₂
ω=0
E₂
C₁
4/3
5/3
2
3
4
8
C₂
0

Fig. 9.

$$\therefore y_1 = y_1^1, \quad \therefore \sin\theta_1 \cos\theta_1 = \sin\theta_1^1 \cos\theta_1^1,$$
$$\therefore \sin 2\theta_1 = \sin 180° - 2\theta_1^1,$$
$$\therefore \theta_1 + \theta_1^1 = 90°, \text{ and } \theta_2 + \theta_2^1 = 90° \qquad (11)$$
$$\therefore \sin\theta_1 = \cos\theta_1^1, \text{ and } \cos\theta_1 = \sin\theta_1^1 \text{ etc.}$$

By eq. (9), $x + x^1 + a_1 + a_2 =$
$$a_1 \cos^2\theta_1 + a_2 \sin^2\theta_1 + a_1 \cos^2\theta_1^1 + a_2 \sin\theta_1^1$$
$$\therefore x + x^1 = 0 \ \therefore x = -x_1 \qquad (12)$$

from which it appears that P^1 is as far to the right of C as P is to the left of it. Hence the loci of P^1 and P correspond point by point in case eq. (11) holds. But in that case G and G^1 have numerically reciprocal values, for we then have

$$\tan(\theta_1 + \theta_1^1) = \frac{\tan\theta_1 + \tan\theta_1^1}{1 - \tan\theta_1 \tan\theta_1^1} = \infty$$
$$\therefore \tan\theta_1 \tan\theta_1^1 = 1, \text{ and } \tan\theta_2 \tan\theta_2^1 = 1$$
$$\therefore \tan\theta_1 \tan\theta_2 \tan\theta_1^1 \tan\theta_2^1 = 1 \ \therefore \ G\,G = \pm 1, \text{ by eq. (5)}$$

Hence the loci of P and P^1 constructed as in Fig. 8, so that their components have the same wattless currents are equal right and left spires, symmetrically situated with reference to CS, and the values of G and G^1 are numerical reciprocals. In Fig. 8, $G = 2$, $G^1 = \frac{1}{2}$ and $R_1 = \frac{8}{9} R_2$ nearly, and this last causes the upper part of the spire to be the larger.

The fact of the symmetrical disposition of the spires in pairs essentially diminishes the labor of preparing a sheet containing spires for various successive values of G such as is represented in Fig. 9; for, when any one spire has been constructed, its reciprocal spire may be readily traced without further construction. Thus it will be seen that all spires for values of G between infinity and unity lie in the first leaf, and all spires for values of G between unity and zero lie in the second leaf, while $G = 1$ is the edge of junction; $G = \infty$ is the external boundary of the first leaf; and $G = 0$ the external boundary of the second leaf.

When spires have been constructed for a comparatively small number of properly chosen successive values of G, intermediates may be readily interpolated by estimation, and then such a diagram as Fig. 9 furnishes a table of values of I and its lag or lead for a given ratio of the resistances, with a single spire in it for each value of G, the different points along the spire corresponding to different values of the frequency. That part of the total area

where the two leaves overlap furnishes apparent interseetions of the set of spires in which $G > 1$ with the set in which $G < 1$ but the sets are on separate leaves and do not really intersect. Every point of $S\ E_1\ T\ E_2$ is a double point with identically the same current I_1 but produced by very different values of L, K, and ω. The only possible way by which any one of these equal values of $I = O\ P$ could be gradually changed to the other would be by gradual variations of frequency etc., by which P should move to the edge of junction in one leaf and back on the other leaf to the same position. The relative positions of E_1 and E_2 depend upon the ratio of the resistances R_1 and R_2; in case $R_1 > R_2$ then E_1 is situated at the left of C as in Fig. 7, and *vice versa*.

A diagram for the case of $R_2 = \frac{5}{13} R_1$ would be that of Fig. 9 reflected in $O\ Q$, or revolved 180° about $O\ Q$. The same is true of any diagrams for reciprocal values of the ratio of the resistances. It is to be noted that for each value of $G > 1$ there is a mimimum value of $I = O\ P$ less than either a_1 or a_2, while for each value of $G < 1$ there is a maximum value of I greater than either a_1 or a_2. Also for each value of G there is in general a maximum value of both lead and lag other than zero. Moreover there is in general a value of I in consonance of phase with E, *i. e.*, it has neither lead nor lag, which differs in magnitude from both a_1 and a_2, and consequently occurs at a frequency lying between infinity and zero. This value of I is less than either a_1 or a_2 if $G > 1$, but is greater than either a_1 or a_2 if $G < 1$. These statements appear from inspection of Fig. 9. As is evident from what has preceded, the loci which are thus obtained are specially useful in tracing the manner in which the total current through two parallel branches varies as the frequency varies from infinity to zero in case the resistances are given and one branch has in it a condenser of known capacity while the other contains a known self-inductance resistance; and the effect on the total current of variations in the capacity of the condenser or in the self-inductance resistance may also be traced by their help though not so readily as variations of frequency. One convenience possessed by these loci is that as we pass from one value of G to another we take steps that begin with the boundaries of one leaf and end with the boundaries of the other, while the common boundary between them is an intermediate

locus. For, in case $G = \infty$ the locus consists of the two exterior bounding semi-circles of the first leaf, in case $G = O$ the locus is the locus that bounds the second leaf in the same manner, and $G_1 = 1$ is the edge of junction, while intermediate values give intermediate loci. The case of $R_1 = R_2$ is worthy of notice, for in that case the spires become circles as shown in Fig. 10, which truth may be demonstrated as follows: Take E in Fig. 10 as origin of the rectangular co-ordinates x and y, but using the other symbols x_1 etc., with meanings already defined, we have, since in this case $O\,E = a = a_1 = a_2$

$$x + a = (x_1 + x_2) = a\,(\cos^2\theta_1 + \cos^2\theta_2)$$

$$y = y_1 - y_2 = a\,(\sin\theta_1 \cos\theta_1 - \sin\theta_2 \cos\theta_2)$$

$$\therefore x = \tfrac{1}{2}\,a\,(\cos 2\,\theta_1 + \cos 2\,\theta_2)$$

$$y = \tfrac{1}{2}\,a\,(\sin 2\,\theta_1 - \sin 2\,\theta_2)$$

$$\therefore x^2 + y^2 = \tfrac{1}{2}\,a^2\,[1 + \cos 2\,(\theta_1 + \theta_2)] = a^2 \cos^2(\theta_1 + \theta_2)$$

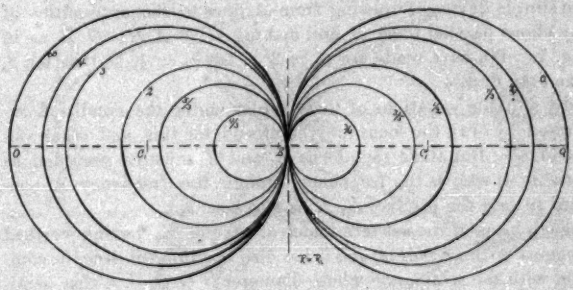


FIG. 10.

$$\text{Also, } \frac{y}{x} = \frac{\sin 2\,\theta_1 - \sin 2\,\theta_2}{\cos 2\,\theta_1 + \cos 2\,\theta_2} = \tan(\theta_1 - \theta_2)$$

$$\therefore \frac{x}{x^2 + y^2} = \cos^2(\theta_1 - \theta_2$$

$$\therefore \frac{a^2\,x^2}{(x^2 + y)_2} = \frac{\cos^2(\theta_1 - \theta_2)}{\cos^2(\theta_1 + \theta_2)} = \left(\frac{1 + \tan\theta_1 \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2}\right)^2 = \left(\frac{1 + G^2}{1 - G^2}\right)^2$$

$$\therefore x^2 + y^2 = \pm\left(\frac{1 - G^2}{1 + G^2}\right) ax \qquad (13)$$

which is a double set of circles having a common tangent at E and lying inside the circles for which G is zero and infinity.

Though the equations of the loci are in this special case expressible in algebraic form in rectangular co-ordinates, such is not generally the case.

5. Consider next the set of loci for the case in which the two branches both contain condensers only, or self-inductions only, in addition to ohmic resistances, and let the product of the tangents of lead (or lag) be assumed constant.

For example in Fig. 11, let

$$\tan \theta_1 \tan \theta_2 = \frac{K_1 K_2}{R_1 R_2 \omega^2} = G^2 \qquad (14)$$

i. e. θ_1 and θ_2 both lie on the same side of $O Q$. If in this case also we take $\tan \theta_1 = G \tan \varphi_1$ and $\tan \theta_2 = G \tan \varphi_2$, then eqs. (6) and (7) likewise hold true. The construction is readily followed in Fig. 11 where the point P on the locus $G = \frac{1}{2}$ is constructed by making θ_1 and θ_2 any two complimentary angles, by the simple device of drawing from A lines to the extremities of any chord parallel to $A B$, and making $G = A B / O B$ as in Fig. 7. We have made $\tan \varphi_1 = 3 \therefore \tan \varphi_2 = \frac{1}{3}$, so that $\tan \theta_1 \frac{2}{3}$ and $\tan \theta_2 = \frac{1}{3}$.

By suitable variations of these angles under the condition expressed by (14), the locus may be traced for this and other values of G. But along each locus K_1 and K_2 must be regarded as variable as well as the frequency, though the frequency is constant in case the product $K_1 K_1$ is also constant.

Each locus of the set terminates at E_1 and E_2, having vertical tangents at those points, and it has also one tangent line in common with the semi-circle whose diameter is $O Q$, *i. e.* that semicircle is the envelope of the set. The point of contact of each locus with this envelope is such that for that point,

$$\tan \theta_1 = \tan \theta_2 = \tan \theta = G \qquad (15)$$

and

$$I = (a_1 + a_2) \cos \theta = \frac{a_1 + a_2}{\sqrt{1 G^2}} \qquad (16)$$

and at this point the locus passes from one leaf to the other.

Those loci for which $G > 1$ have one or more maximum values of I. Others may have a maximum value of I near that given by eq. (16); but for small values of G there is no maximum value of I. For small values of G there may be one or more maximum values of θ. Others may have θ a maximum not far from that given in eq. (16), but for large values of G there is none.

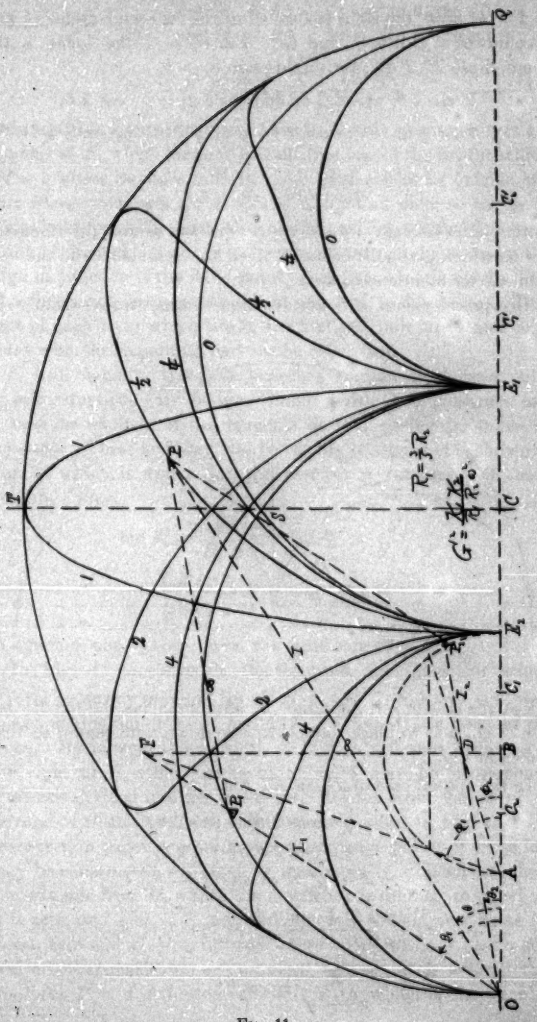

Q
Σ_2
Σ_1
Σ_0
E
C
B
A
O
E_2
E_1
E_0
C_2
C_1
C_0
B_2
B_1
B_0
A_2
A_1
A_0
L_1
L_2
L_0
$R_1 = \frac{1}{2} R_0$
$G = \frac{R_0 X_0}{R_0^2 + X_0^2}$
0
1
2
3
4
5
6
7
8
9
10
11
12

FIG. 11.

For $G = \infty$ the locus is formed of the two semi-circles C_1 and C_2; for $G = 0$ it is C_1^1 and C_2^1. For $G = 1$ the locus is the semi-ellipse $E_1\ T\ E_2$; for, in this case $\theta_1 + \theta_2 = 90°$.

$$\therefore \sin 2\theta_1 = \sin 2\theta_2 \text{ and } \cos 2\theta_1 = -\cos 2\theta_2$$

$$\therefore x = \tfrac{1}{2}(a_1 - a_2)\cos 2\theta_1,\ y = \tfrac{1}{2}(a_1 + a_2)\sin 2\theta_2$$

$$\therefore \frac{x^2}{(a_1 - a_2)^2} + \frac{y^2}{(a_1 + a_2)^2} = \tfrac{1}{4} \qquad (17)$$

For intermediate values of G the loci assume intermediate forms which gradually change from the semi-ellipse to the two semi-circles mentioned above.

Reciprocal values of G are disposed symmetrically about $S\ T$.

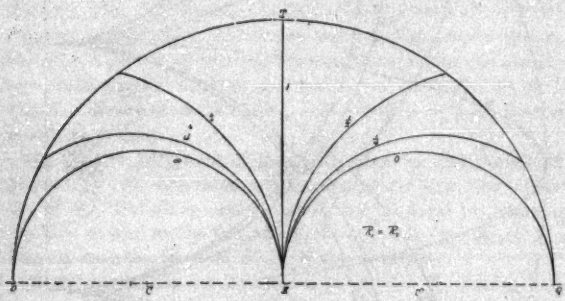


Fig. 12.

For the case of self-induction in both branches the loci lie below $O\ Q$, so as to place the loci upside down with regard to that shown in Fig. 11.

In the special case of $R_1 = R_2$, Fig. 12

$$x = \tfrac{1}{2}a(\cos 2\theta_1 + \cos 2\theta_2)$$

$$y = \tfrac{1}{2}a(\sin 2\theta_1 + \sin 2\theta_2)$$

$$\therefore x^2 + y^2 = a^2 \cos^2(\theta_1 - \theta_2)$$

$$\frac{y}{x} = \frac{\sin 2\theta_1 + \sin 2\theta_2}{\cos 2\theta_1 + \cos 2\theta_1} = \tan(\theta_1 + \theta_2).$$

$$\therefore \frac{x^2}{x^2 + y^2} = \cos^2(\theta_1 + \theta_2)$$

$$\therefore \frac{a^2 x^2}{(x^2 + y^2)^2} = \frac{\cos^2(\theta_1 + \theta_2)}{\cos^2(\theta_1 - \theta_2)} = \left(\frac{1 - \tan\theta_1 \tan\theta_2}{1 + \tan\theta_1 \tan\theta_2}\right)^2$$

$$\therefore x^2 + y^2 = \pm \left(\frac{1 + G^2}{1 - G^2}\right) a\, x \qquad (18)$$

This equation represents a set of circles having a common vertical tangent at E, which tangent is itself that one of the set for which $G = 1$ whose radius is infinity. All these circles lie outside the pair whose diameters are $O\,E$ and $Q\,E$, and only so much of each circle forms the locus in question as lies within the semi-circle of diameter $O\,Q$. The points where each locus meets the edge of junction is the fixed point of contact mentioned in Fig. 11, where each locus passes from one leaf to the other. That part of each locus which lies in the first leaf falls upon and coincides with the remaining part in the other leaf.

6. Still assuming that both branches contain condensers only, or self-inductions only besides ohmic resistances, construct loci in case the quotient of the tangents of lead or lag be taken as constant. These loci may also be readily constructed as shown in Fig. 13 which is drawn for the case of a condenser in each branch. Then

$$\tan\theta_1 = \frac{K_1}{R_1\,\omega} \text{ and } \tan\theta_2 = \frac{K_2}{R_2\,\omega}$$

$$\therefore \frac{\tan\theta_1}{\tan\theta_2} = \frac{K_1 R_2}{K_2 R_1} = F, \text{ say} \qquad (19)$$

in which θ_1 and θ_2 both lie on the same side of $O\,C$.

In Fig. 13 to construct, for example, the locus for which $F = 4$:- at any point B lay off $B\,F / B\,D = 4$ and complete the parallelogram on $O\,P_1$ and $O\,P_2$. Then P is a point of the locus. By moving this vertical $B\,D\,F$ to different distances to the right of O, other points of $F = 4$ may be constructed. Verticals divided in other ratios give loci for other values of F. Several of these have been constructed by points in Fig. 13. On cross-section paper or profile paper successive points of these loci may be constructed with facility and rapidity. Each of these loci extends from O, where the frequency is infinite, to Q, where it is zero; and $I = O\,P$ may consequently assume any value between zero and $a_1 + a_2$ for each value of F, while I has in general no other maximum or minimum values. It is evident that the locus $F = I$ is the semi-circle $O\,T\,Q$, which is the edge of

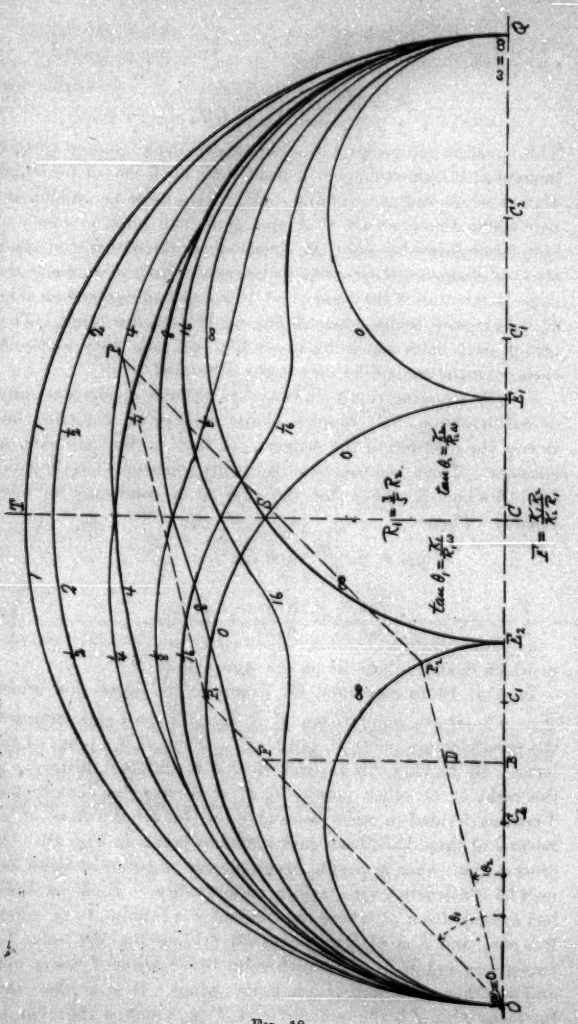

Q
ω = ∞
C'₂
C'₁
E₁
C'
T
C₂
C₁
E₂
E
D
O
$R_1 = \frac{1}{2} R_2$
$\tan\theta_1 = \frac{X_1}{R_1}$
$\tan\theta = \frac{X}{R}$
$T = \frac{X_1 R_2}{X R_1}$

FIG. 13.

junction along which the two leaves join. The system of loci lying in the first leaf are those whose values of F lie between infinity and unity, and they constitute a series of curves which gradually change in shape from the two semi-circles $O\ E_2$ and $E_2\ Q$ in case $F = \infty$, to the single semi-circle $O\ T\ Q$ in case $F = 1$.

To each of these curves in the first leaf there corresponds an equal locus in the second leaf, such that the value of F is the numerical reciprocal of that in the first leaf F, or, if we distinguish magnitudes in the second leaf by primes and let the components I_1^1 and I_2^1 have such directions that

$$\tan\theta_1 \tan\theta_1^1 = 1, \text{ and } \tan\theta_2 \tan\theta_2^1 = 1,$$

then

$$\frac{\tan\theta_1 \tan\theta_1^1}{\tan\theta_2 \tan\theta_2^1} = F_1 F_1^1 = 1\text{; but } \sin\theta_1 = \cos\theta_1^1, \text{ etc.,}$$

and

$$\cos\theta_1 = \sin\theta_1^1, \text{ etc.} \qquad (20)$$

Hence, as shown in eq. (12) P^1 is as far to the right of C as P is to the left of it.

But, $\quad y = y_1 + y_2 = a_1 \sin\theta_1 \cos\theta_1 + a_2 \sin\theta_2 \cos\theta_2$

and $\quad y_1 = y_1^1 + y_2^1 = a_1 \sin\theta_1^1 \cos\theta_1^1 + a_2 \sin\theta_2^1 \cos\theta_2^1$

$\therefore$ by eq. (20), $y = y^1$, or P and P^1 are at the same distance from $O\ Q$. Hence, in case $F\ F^1 = 1$ the loci of P and P^1 are pairs of equal curves symmetrically situated about $C\ T$.

Each locus of the set $F > 1$ apparently intersects each of the set $F < 1$, at some point in the area $O\ S\ Q\ T$, but such is not really the case, since all of the first set lie in the first leaf, while all of the other set lie in the second leaf which overlaps the first in the area just mentioned; so that of two coincident values of $I = O\ P$ which lead to overlapping points, or apparently coincident points, the one can be gradually and continuously changed into the other only by moving P so as to cause F to assume all intermediate values, including $F = 1$.

For very large or very small values of F the locus approximates to the semi-circles with centers $C_2\ C_2^1$ or to $C_1\ C_1^1$. It will be noticed in these cases that since the curve dips toward E_2 or E_1 there may be one value of I for which the lead is a minimum, and another for which it is a maximum, and these are to be located by drawing tangents to the curve through 0; but values of F that are neither large nor small afford no maximum or minimum lead except 0° and 90°.

In case both branches contain self-inductive resistances instead of condensers the diagram lies below $O\ Q$ instead of above it.

There is always one value of I for each value of F that affords a maximum wattless current, and for very large and very small values of F there may be two such maxima and one minimum between them. These can be located by drawing horizontal tangents to the locus, or they may be determined analytically by the conditions for a maximum or minimum of y.

Every value of I has a lead, and hence no case of consonance of phase between E and I occurs for any finite frequency, but that state is more nearly possible the more nearly F approaches to zero or infinity. Each locus becomes more and more nearly symmetrical about $S\ T$ the more R_1 and R_2 approach equality, and the special case of $R_1 = R_2$ causes each pair of loci for reciprocal values of F to coincide, *i. e.*, each locus is symmetrical about $S\ T$. The equation of the locus in rectangular co-ordinates may then be obtained in algebraic form. Take the origin at C, then using expressions found previously we have: $a_1 = a_2 = a$, and $x = \frac{1}{2}\ a\ (\cos 2\ \theta_1 + \cos 2\ \theta_2)$, $y = a\ (\sin\ 2\ \theta_1 + \sin\ 2\ \theta_2)$

$$\therefore x^2 + y^2 = a^2 \cos^2 (\theta_1 - \theta_2).$$

$$a^2 - x^2 - y^2 = a^2 \sin^2 (\theta_1 - \theta_2)$$

Also,

$$\frac{y}{x} = \frac{\sin 2\ \theta_1 + \sin 2\ \theta_2}{\cos 2\ \theta_1 + \cos 2\ \theta_2} = \tan (\theta_1 + \theta_2)$$

$$\therefore \frac{1}{1 + \frac{x^2}{y^2}} = \frac{y^2}{x^2 + y^2} = \sin^2 (\theta_1 + \theta_2)$$

$$\therefore \frac{a^2\ y^2}{(x^2 + y^2)\ (a^2 - x^2 - y^2)} = \frac{\sin^2 (\theta\ + \theta_2)}{\sin^2 (\theta_1 - \theta_2)} = \left(\frac{\tan \theta_1 + \tan \theta_2}{\tan \theta_1 - \tan \theta_2}\right)^2$$

$$\therefore (x^2 + y^2)\ (a^2 - x^2 - y^2) = \left(\frac{F - 1}{F + 1}\right)^2 a^2\ y^2 \qquad (21)$$

a curve of the fourth degree which is symmetrical about both x and y, of which the upper half is to be used in case of condensers, and the lower half for self-inductance resistances. Equation (21) includes both cases.

The general case in Fig. 13 is specially suited to treat variable frequency, for F remains unchanged by variation of frequency. It will be unnecessary to more than mention in passing the case in which one of the resistances is very small, and the locus is then confined to a narrow band along the semi-circle $O\ T\ Q$.

7. Resuming the case of a condenser in the first branch and self-induction in the second, construct the loci which make the ratio of the curvilinear co-ordinates constant, *i. e.*,

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{K R_2}{L R_1 \omega^2} = F, \text{ say} \tag{22}$$

In Fig. 14 take $F = 4$, for example, then $\overline{B F} / \overline{B D} = 4$ and the construction proceeds as in Fig. 13 except in this that $B F$ and $B D$ being opposite in sign lie on opposite sides of $O Q$. The loci for various values of F appear on Fig. 14, As before stated, the process of constructing by points is much facilitated by using paper ruled in squares. Each of these loci extends from O, where the frequency is infinite, to Q where it is zero, and the current $I = O P$ may consequently have any value between zero and $a_1 + a_2$ for each value of F, while I has in general no other maximum or minimum values. But each locus is tangent at one point to the edge of junction $E_1 T E_2$ and passes at that point from one leaf to the other, so that the envelope of the loci, as F varies from infinity to zero, is the semi-circle of radius $\pm \frac{1}{2}(a_1 - a_2)$ and center C.

It may be proven by steps like those previously given that in case $F F' = 1$ (so that F and F' are numerically reciprocal), these loci are equal right and left curves symmetrically disposed about $S T$. Thus the loci are separated into two sets according as F is greater or less than unity.

The locus for $F = 1$ is its own reciprocal and consequently its right and left half are alike. It is in fact a semi-ellipse $O T Q$ as may readily be proved. For, in this case $\theta_1 = \theta_2$, and taking the origin at C we have by using the same notation as previously

$$x + \tfrac{1}{2}(a_1 + a_2) = x_1 + x_2 = (a_1 + a_2) \cos^2 \theta_1$$

$$y = y_1 - y_2 = (a_1 - a_2) \sin \theta_1 \cos \theta_1$$

$$\therefore x = \tfrac{1}{2}(a_1 + a_2) \cos 2\theta_1, \quad y + \tfrac{1}{2}(a_1 - a_2) \sin 2\theta_1$$

$$\therefore \frac{x^2}{(a_1 + a_2)^2} + \frac{y^2}{(a_1 - a_2)^2} = \tfrac{1}{4} \tag{23}$$

This ellipse is tangent at T to the semi-circle of center C and radius $\pm \frac{1}{2}(a_1 - a_2)$, while all those loci for which $F > 1$ are tangent to it between E_2 and T, and all those for which $F < 1$ between E_1 and T. The apparent intersections of the loci of the two sets within the area $S E_1 T E_2$ are double points situated in different leaves.

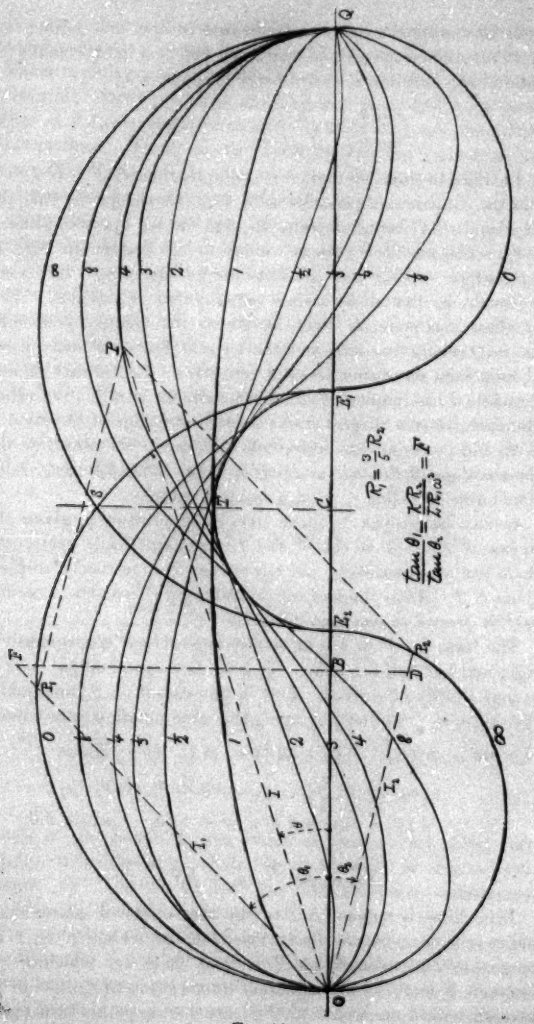

Q
O
P
P₁
F
F₁
F₂
A
B
C
D
$R_1 = \frac{3}{5} R_2$
$\frac{\tan\theta_1}{\tan\theta_2} = \frac{KR_2}{KR_1\omega^3} = F$

Fig. 14.

For each value of $F > 1$ there is in general a value of I between $\frac{1}{2}(a_1 - a_2)$ and $(a_1 + a_2)$ at which the lead is a maximum and for each value of $F < 1$, a maximum lag between the same limits, which are to be located by drawing tangents to the loci through O.

There are also maximum wattless currents to be located by drawing horizontal tangents to the loci. There is also for each locus a current I in consonance with E determined by the point where it crosses $O\,Q$, viz: $I < a_2$ in case $F > 1$, and $I > a_1$ in case $F < 1$.

In the special case of $R_1 = R_2$ the circular envelope reduces to the point C through which each locus passes. In this case the loci all fall within two circles whose diameters are $C\,O$ and $C\,Q$, and the locus for any given value of F consists of two parts which are exactly alike (one in each circle) and situated opposite to each other. Thus the pair of loci for any given value of F and its numerical reciprocal form a figure ∞, whose equation is found as follows:—Since in this case $a_1 = a_2 = a$ as before.

$$x = \frac{a}{2}(\cos 2\,\theta_1 + \cos 2\,\theta)$$

$$y = \frac{a}{2}(\sin 2\,\theta_1 - \sin 2\,\theta_2)$$

$$\therefore x^2 + y^2 = a^2 \cos^2(\theta_1 + \theta_2)$$

$$\therefore a^2 - x^2 - y^2 = a^2 \sin^2(\theta_1 + \theta_2)$$

also

$$\frac{y}{x} = \frac{\sin 2\,\theta_1 - \sin 2\,\theta_2}{\cos 2\,\theta_1 + \cos 2\,\theta_2} = \tan(\theta_1 - \theta_2)$$

$$\therefore \frac{y^2}{x^2 + y^2} = \sin^2(\theta_1 - \theta_2)$$

$$\therefore \frac{(a^2 - x^2 - y^2)(x^2 + y^2)}{a^2\,y^2} = \frac{\sin^2(\theta_1 + \theta_2)}{\sin^2(\theta_1 - \theta_2)} = \left(\frac{\tan\theta_1 + \tan\theta_2}{\tan\theta_1 - \tan\theta_2}\right)^2$$

$$\therefore (a^2 - x^2 - y^2)(x^2 + y^2) = \left(\frac{F+1}{F-1}\right)^2 a^2\,y^2 \qquad (24)$$

which is evidently symmetrical about both the axes of x and y, and is the same for reciprocal values of F. It is not thought necessary to show a figure for this case. It will be noticed that equations (21) and (24) are essentially the same, provided F may have any values from positive to negative infinity. Were θ_1 and

θ_2 to be always reckoned positive for lead and negative for lag or *vice versa*, then F would have this range of values. It has been restricted thus far in this paper to positive values by our conventions of lead and lag. These equations reduce to a circle of radius a in case $F = 1$, and to two circles tangent to $C\,S$ at C in case F is zero or infinity.

8. Consider next the more general case in which each branch contains an ohmic resistance, a self-inductive resistance and a

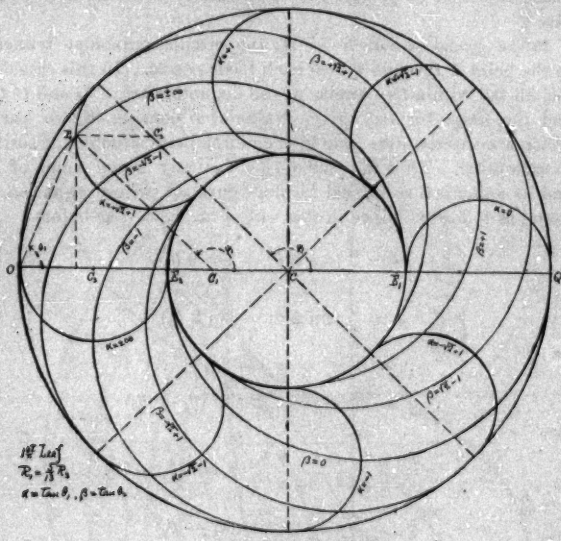


FIG. 15.

condenser, but no mutual induction. Then using notation like that before employed we have the known equations.:

$$\tan \theta_1 = \frac{K_1}{R_1 \omega} - \frac{L_1 \omega}{R_1}, \; I_1 = \frac{E}{R_1} \cos \theta_1 \qquad (25)$$

$$\tan \theta_2 = \frac{K_2}{R_2 \omega} - \frac{L_2 \omega}{R_2}, \; I_2 = \frac{E}{R_2} \cos \theta_2 \qquad (26)$$

in which θ_1 and θ_2 are angles of lead or lag according as they are positive or negative; and they may evidently either of them be

made to assume any value between + 90° and — 90° (while R_1 and R_2 remain constant), either by suitably varying the frequency or the other constants of the branches.

Since I_1 and I_2 vary in this case as the cosines of θ_1 and θ_2, as also in equations (2) and (4), it is evident that the loci for constant values of tan θ_1 or tan θ_2 are circular arcs as in Figs. 4 and 5; but from the fact that the ranges of θ_1 and θ_2 no longer lie between 0 and 90° but between + 90° and — 90° instead, it ap-

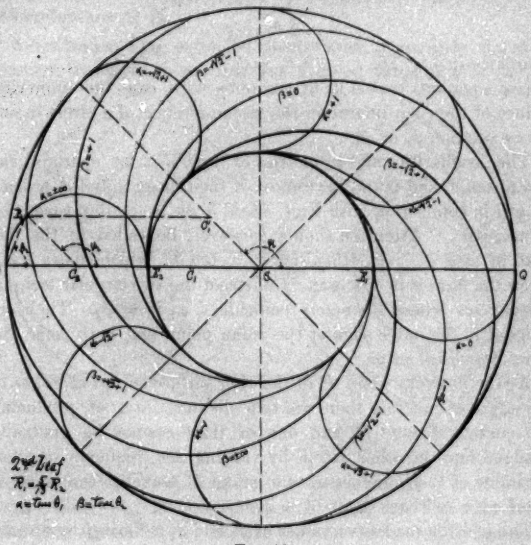


FIG. 16.

pears that the loci are no longer semi-circles merely as in Figs. 4 and 5, but each includes in this case an entire circumference, one-half of which lies in each leaf.

In Figs. 15 and 16 these leaves have been for convenience and clearness drawn separately, and it is evident that every point P (the extremity of the vector $I = O\,P$) lies within the area included between the two circumferences whose diameters are $E_1\,E_2$ and $O\,Q$. On this two-leaved surface each circle of one

set has one intersection with each circle of the other set on each leaf, and the two leaves are united along the exterior and interior circumferences forming a flat ring with two faces.

The positions of the various loci may be most conveniently designated by the angular positions of their centers with reference to the line $O\ Q$. For example, in Fig. 15, let $Q\ C\ C_2^1 = Q\ C_1\ P_1 = \phi_1$ then we have from the figure

$$\tan\theta_1 = \frac{\sin(180^\circ - \phi_1}{1 - \cos(180^\circ - \phi_1} = \frac{\sin\phi_1}{1 + \cos\phi_1} = \tan\tfrac{1}{2}\phi_1 \qquad (27)$$

$$\therefore 2\,\theta_1 = \phi_1$$

Similarly in Fig. 16, $2\,\theta_2 = \phi_2$.

These equations enable us to compute with ease the numerical values of the co-ordinates on the circumference of a circle in any given position ϕ_1 or ϕ_2.

The peculiarities and properties of these loci for constant values of $\tan\theta_1$ and $\tan\theta_2$ have most of them been detailed at such length in connection with Figs. 4 and 5 as to render repetition unnecessary. Attention should, however, be called to the fact that in case $\phi_1 = \phi_2$, then $\tan\theta_1 = \tan\theta_2$, which shows itself upon the figures in this way: Construct the two circular loci one of each set whose diameters coincide, *i. e.*, $\phi_1 = \phi_2$. They are tangent to the outer edge at the same point and both have the same numerical value.

Again, at every point P of the ring surface except at either of the edges of junction there are two distinct ways of producing the current $I = O\ P$, and one of these cannot be gradually changed into the other except by varying the frequency or other constants of the branches so as to make P travel to the inner or outer edge and back again.

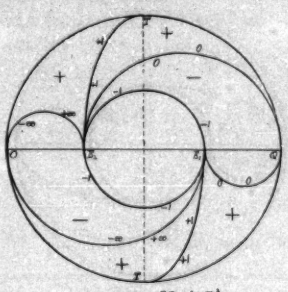
The loci for successive values of ϕ_1 and ϕ_2 differing by 45° are represented upon Figs. 15 and 16.

9. In continuing the consideration of the general case, expressed by eqs., (25) and (26), let G^2 designate the product of the co-ordinates, as was done previously, *i. e.* $\tan\theta_1 \tan\theta_2 = G^2$. Then the loci for various constant values of G will be those already discussed, but by reason of the doubling of range of θ_1 and θ_2 the loci will combine together the various separate results previously discussed, as shown in Figs. 17 and 18, where the two leaves are shown separately. In these figures, angles of lead are taken as positive, and lag as negative, giving as a result negative

products whenever the a lead and a lag are combined. For all negative values of G^1, *i. e.* $-\infty < G < 0$, the loci lie in a single leaf and join the points E_1 and E_2 of the same leaf, as in Fig. 9. But for all positive values of G^2, *i. e.* $0 < G^2 < \infty$ the loci join E_1 of one leaf to E_1 of the other leaf as in Fig. 11.

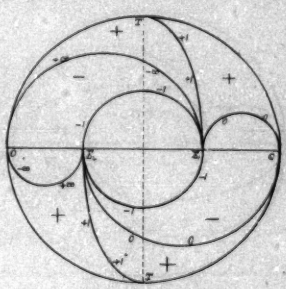
The manner in which these loci succeed one another is clear from Figs. 17 and 18, from which it appears that the values of G^2 along the circumference $O\ T\ Q$ are all positive, being infinity at O, unity at T, and zero at Q, while $G^3 = -1$ along the entire circumference $E_1\ E_2$.

Since lead and lag enter the construction in precisely the same manner, the locus in one leaf for a given value of G is the re-



1st Leaf, G^2

FIG. 17.



2nd Leaf, G^2

FIG. 18.

flection in $O\ Q$ of the locus for the same value of G in the other leaf. Thus Fig. 17 is a reflection in $O\ Q$ of Fig. 18. It has also been shown before, that the loci for reciprocal values of G are reflections of one another in $T\ T$. These last reflections are in different leaves. By the time a locus has suffered reflections successively in $O\ Q$ and $T\ T$ it has been restored to the leaf from which it started and has been revolved 180° about the center of the leaves. Hence, if from any point P we draw a line $P\ C$ and prolong it to P^1 so that $C\ P = C\ P^1$, then for these two points we have $G\ G^1 = 1$.

10. In like manner consider the loci in which the quotient $\tan\theta_1 / \tan\theta_2 = F$ assumes various constant values in the general case of eqs. (25) and (26).

These will consist of combinations of the special cases of Figs. 13 and 14 as shown in separate leaves in Figs. 19 and 20 where the blank spaces may be thought of as filled in with loci for successive values of F from Figs. 13 and 14, due regard being had to their signs. $F = +1$ for the entire circumference $O\ Q$, but F is negative along $E_2\ S\ E_1$; $-\infty$ at E_2, -1 at S, and 0 at E_1.

The same statements respecting reflections etc. are true of the F — loci as of the G — loci, hence if $C\ P = C\ P^1$ and $P\ C\ P^1$ is a straight line, then $F\ F^1 = 1$.

11. In further continuation of the general case in eqs. (25) and (26), it is now proposed to develop the general graphical

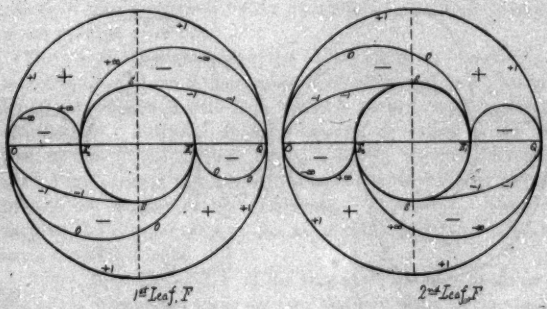


Fig. 19. Fig. 20.

process for constructing the locus of the extremity P of the vector $I = O\ P$ representing the total current in case the frequency alone is available.

In eq. (25) let the subscript 1 designate magnitudes in the first branch: viz, let

$$\tan\ \theta_1' = \frac{K_1}{R_1\ \omega}, \text{ and } \tan\ \theta_1'' = \frac{L_1\ \omega}{R_1}$$

$$\therefore \tan\ \theta_1 = \tan\ \theta_1' - \tan\ \theta_1''$$

and let,

$$\tan\ \theta_1' \tan\ \theta_1'' = \frac{K_1\ L_1}{R_1^2} = G_1^2$$

Also let,

$$\tan \theta_1' = G_1 \tan \varphi_1', \text{ and } \tan \theta_1'' = G_1 \tan \varphi_1''$$

$$\therefore \tan \varphi_1' \tan \varphi_1'' = 1, \ \therefore \varphi_1' + \varphi_1'' = 90°.$$

In like manner let magnitudes in the second branch be designated by the subscript 2.

Now in Fig. 21 take any convenient distances $B\,F$ and $B\,O_1$ and let $B\,O_1\,F = \theta_1'$,
then,

$$\tan\ \theta_1' = \frac{K_1}{R_1\,\omega} = \frac{B\,F}{B\,O_1} \tag{28}$$

Whatever may be the values of the constants K_1 and R_1 in this branch, and whatever distances $B\,F$ and $B\,O_1$ may have been assumed, a value of ω which corresponds to them, can be found from eq. (28) if need be.

Next, lay off

$$B\,A_1 = G_1\,\overline{B\,O_1}$$

$$\therefore \tan \theta_1' = \frac{B\,F}{B\,O_1} = G_1 \frac{B\,F}{B\,A_1} = G \tan \varphi_1'$$

$$\therefore B\,A_1\,F = \varphi_1'$$

Make $F\,A_1\,D_1 = 90°$, then $B\,A_1\,D_1 = \varphi_1''$

$$\therefore B\,O_1\,D_1 = \theta_1'' \text{ and } \tan \theta_1 = \frac{B\,D_1}{B\,O_1}$$

$$\therefore \tan \theta_1 = \tan \theta_1' - \tan \theta_1'' = \frac{B\,F - B\,D_1}{B\,O_1}$$

Lay off

$$B\,H_1 = B\,F - B\,D_1, \ \therefore \tan \theta_1 = \frac{B\,H_1}{B\,O_1} \text{ and } \theta_1 = B\,O_1\,H_1$$

Again, lay off $B\,O_2$ such that

$$\frac{R_1}{K_1} : \frac{R_2}{K_2} :: B\,O_1 : B\,O_2,$$

and in this proportion substitute the value of $B\,O_1$ from eq. (28).

$$\tan \theta_2' = \frac{K_2}{R_2\,\omega} = \frac{B\,F}{B\,O_2} \tag{29}$$

Complete the construction for this branch as in the first branch and we find

$$\theta_2' = B\,O_2\,F, \ \theta_2'' = B\,O_2\,D_2, \ \theta_2 = B\,O_2\,H_2$$

In Fig. 21, the constants were so assumed that $G_1 = \frac{1}{2}$ and $G_2 = 2$.

We have thus found θ_1 and θ_2, the angles of lead (or lag) of the components I_1 and I_2 for some unknown value of ω_1 which could be readily computed from eq. (28) or (29), if desired for any purpose. Having thus found the relative positions of O_1 O_2, A_1

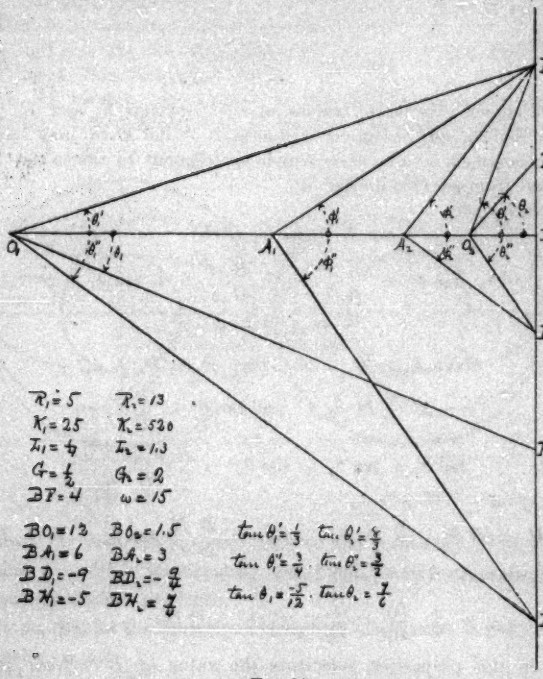


FIG. 21.

A_2, the point F may be moved to successive positions on the vertical through B. Each successive position of F will give values of θ_1 and θ_2 which correspond to some one frequency, and thus completely determine the components I_1 and I_2 of the total current I, for that undetermined frequency.

The construction just given is one of two alternative constructions, for it would have been equally possible to assume at the start any two distances $B\ O_1$ and $B\ D$ on the vertical below B and then let

$$\tan \theta_1'' = \frac{L_1 \omega}{R_1} = \frac{B\ D}{B\ O_1}$$

and

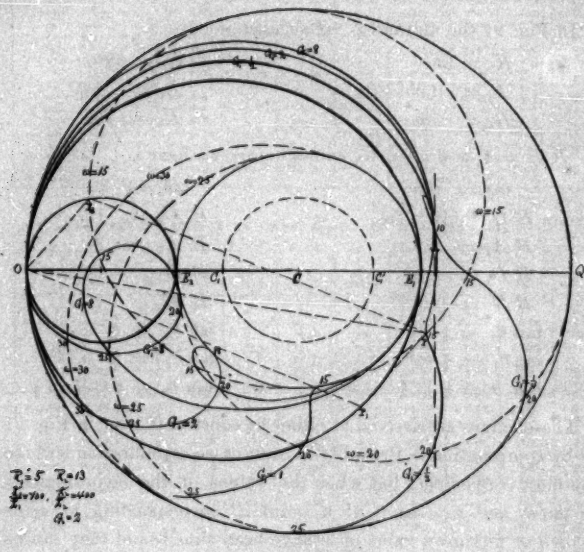


FIG. 22

$$B\ O_1 : B\ O_2 :: \frac{R_1}{L_1} : \frac{R_2}{L_2}$$

$$\therefore \tan \theta_2'' = \frac{L_2 \omega}{R_2} = \frac{B\ D}{B\ O_2} :$$

Then completing the construction in a manner similar to that just given find two points F_1 and F_2 from which H_1 and H_2

could be found. Successive positions of D would in this case thus serve to determine corresponding values of θ_1 and θ_2 precisely as they were determined in the previous construction by successive positions of F. It may often be more expeditious to compute numerical values of $\tan \theta_1$ and $\tan \theta_2$ directly from eqs. (25) and (26) by assuming various successive values of ω, and then, if the locus of P be constructed on cross-section paper, θ_1 and θ_2 can be laid off with facility, since $\tan \theta_1$ and $\tan \theta_2$ are ratios such as can be read off at once on squared paper.

In Fig. 21 the numerical values assumed are:

$K_1 = 25.$	$K_2 = 520.$
$L_1 = 0.25$	$L_2 = 1.3$
$R_1 = .5$	$R_2 = 13.$

$B\,F = 4$, and $\omega = 15$, consequently we have:

$G_1 = 0.5$	$G_2 = 2.$
$B\,O_1 = 12.$	$B\,O_2 = 1.5$
$B\,A_1 = 6.$	$B\,A_2 = 3.$
$B\,D_1 = -9.$	$B\,D_2 = -2.25$
$B\,H_1 = -5.$	$B\,H_2 = 1.75$
$\tan \theta_1' = \frac{1}{3}$	$\tan \theta_2' = \frac{3}{8}$
$\tan \theta_1'' = \frac{3}{4}$	$\tan \theta_2'' = \frac{3}{8}$
$\tan \theta_1 = \frac{1}{3} - \frac{3}{4} = -\frac{5}{12}$	$\tan \theta_2 = \frac{3}{8} - \frac{3}{8} = \frac{7}{6}$

These values are arrived at either by construction as in Fig. 21 or by computation. Probably in most cases computation will be the more convenient; but when the values of the co-ordinates $a = \tan \theta_1$ and $\beta = \tan \theta_2$ of a point P corresponding to some known or unknown value of ω have been thus found they can be at once used as in Fig. 22 for plotting the point P of the locus for $G_1 = \frac{1}{3}$ and $G_2 = 2$.

Numerical values from which the curves in Fig. 22 were plotted, are given in the following tables of values of

$$a = \frac{K_1 - L_1 \omega^2}{R_1 \omega} \qquad \beta = \frac{K_2 - L_2 \omega^2}{R_2 \omega}$$

$$G_1 = \frac{\sqrt{K_1 L_1}}{R_1} \qquad G_2 = \frac{\sqrt{K_2 L_2}}{R_2}$$

VALUES OF G_1 AND α, IN CASE $R_1 = 5$ AND $K_1 / L_1 = 100$.

$K_1 =$	12.5	25.	50.	100.	200.	18.75	37.5	75.	150.	300.
$L_1 =$	0.125	0.25	0.5	1.	2.	.1875	0.375	0.75	1.5	3.
$G_1 =$	0.25	0.5	1.	2.	4.	0.375	0.75	1.5	3.	6.
$\omega = 1$	2.47	4.95	9.9	19.8	39.6	3.78	7.45	14.9	29.7	59.4
4	0.52	1.05	2.1	4.2	8.4	0.79	1.57	3.15	6.3	12.6
5	0.37	0.75	1.5	3.	6.	0.56	1.12	2.25	4.5	9.
6	0.26	0.53	1.06	2.13	4.26	0.4	0.8	1.59	3.18	6.36
7	0.19	0.37	0.75	1.5	3.	0.28	0.56	1.12	2.25	4.5
8	0.11	0.22	0.45	0.9	1.8	0.17	0.34	0.68	1.35	2.7
9	0.05	0.11	0.21	0.42	0.84	0.08	0.16	0.32	0.63	1.26
10	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
11	— 0.05	— 0.09	— 0.18	— 0.36	— 0.72	— 0.07	— 0.14	— 0.27	— 0.54	— 1.08
12	— 0.09	— 0.18	— 0.36	— 0.73	— 1.47	— 0.14	— 0.27	— 0.54	— 1.08	— 2.16
13	— 0.13	— 0.27	— 0.53	— 1.06	— 2.12	— 0.2	— 0.4	— 0.8	— 1.59	— 3.18
14	— 0.17	— 0.34	— 0.69	— 1.37	— 2.74	— 0.26	— 0.52	— 1.04	— 2.07	— 4.14
15	— 0.21	— 0.42	— 0.83	— 1.67	— 3.33	— 0.31	— 0.62	— 1.25	— 2.5	— 5.
16	— 0.24	— 0.48	— 0.97	— 1.93	— 3.9	— 0.37	— 0.73	— 1.46	— 2.91	— 5.82
17	— 0.28	— 0.56	— 1.11	— 2.22	— 4.45	— 0.42	— 0.83	— 1.67	— 3.33	— 6.66
18	— 0.31	— 0.62	— 1.25	— 2.49	— 4.98	— 0.47	— 0.94	— 1.87	— 3.74	— 7.48
19	— 0.34	— 0.68	— 1.37	— 2.75	— 5.5	— 0.52	— 1.01	— 2.06	— 4.12	— 8.24
20	— 0.37	— 0.75	— 1.5	— 3.	— 6.	— 0.56	— 1.12	— 2.25	— 4.5	— 9.
25	— 0.52	— 1.05	— 2.1	— 4.2	— 8.4	— 0.79	— 1.57	— 3.15	— 6.3	— 12.6
30	— 0.67	— 1.35	— 2.5	— 5.	— 10.	— 0.94	— 1.88	— 3.75	— 7.5	— 15.
40	— 0.94	— 1.87	— 3.75	— 7.5	— 15.	— 1.41	— 2.81	— 5.62	— 11.25	— 22.5
60	— 1.45	— 2.9	— 5.8	— 11.6	— 23.3	— 2.68	— 4.35	— 8.7	— 17.4	— 34.8
80	— 2.	— 4.	— 8.	— 16.	— 32.	— 3.	— 6.	— 12.	— 24.	— 48.
100	— 2.47	— 4.95	— 9.9	— 19.8	— 39.6	— 3.73	— 7.45	— 14.9	— 29.7	— 59.4

VALUES OF G_2 AND β, IN CASE $R_2 = 13$ AND $K_2 / L_2 = 400$.

$K_2 =$	65.	130.	260	520.	1040	97.5	195.	390.	780.	1560
$L_2 =$	.1625	0.325	0.65	1.3	2.6	.24375	.4875	0.975	1.95	3.9
$G_2 =$	0.25	0.5	1.	2.	4.	0.375	0.75	1.5	3.	6.
$\omega = 1$	4.99	9.97	19.95	39.9	79.8	7.48	14.96	29.92	59.85	119.7
4	1.2	2.4	4.8	9.6	19.2	1.8	3.6	7.2	14.4	28.8
5	0.94	1.87	3.75	7.5	15.	1.4	2.81	5.62	11.25	22.5
6	0.7	1.39	2.78	5.55	11.1	1.05	2.09	4.17	8.34	16.68
7	0.63	1.25	2.5	5.	10.	0.94	1.87	3.75	7.5	15.
8	0.53	1.05	2.1	4.2	8.4	0.79	1.57	3.15	6.3	12.6
9	0.45	0.9	1.8	3.6	7.1	0.68	1.35	2.7	5.4	10.8
10	0.38	0.75	1.5	3.	6.	0.56	1.12	2.25	4.5	9.
11	0.34	0.68	1.27	2.55	5.11	0.48	0.95	1.9	3.8	7.62
12	0.27	0.53	1.07	2.15	4.3	0.4	0.8	1.6	3.21	6.42
13	0.22	0.44	0.88	1.77	3.55	0.3	0.61	1.22	2.43	4.86
14	0.18	0.36	0.73	1.46	2.92	0.28	0.55	1.09	2.19	4.38
15	0.15	0.29	0.58	1.16	2.33	0.22	0.44	0.87	1.74	3.48
16	0.11	0.22	0.45	0.9	1.8	0.16	0.31	0.62	1.25	1.5
17	0.08	0.16	0.32	0.65	1.3	0.12	0.24	0.48	0.96	1.92
18	0.05	0.1	0.21	0.42	0.84	0.8	0.15	0.31	0.63	1.26
19	0.03	0.05	0.1	0.2	0.41	0.04	0.07	0.15	0.3	0.6
20	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
25	— 0.11	— 0.22	— 0.45	— 0.9	— 1.8	— 0.17	— 0.34	— 0.67	— 1.35	— 2.7
30	— 0.21	— 0.41	— 0.83	— 1.67	— 3.33	— 0.32	— 0.63	— 1.24	— 2.49	— 4.98
40	— 0.38	— 0.75	— 1.5	— 3.	— 6.	— 0.69	— 1.37	— 2.75	— 4.5	— 9.
60	— 0.66	— 1.32	— 2.65	— 5.3	— 10.6	— 0.99	— 1.99	— 3.97	— 7.95	— 15.9
80	— 0.94	— 1.87	— 3.75	— 7.5	— 15.	— 1.4	— 2.81	— 5.62	— 11.25	— 22.5
100	— 1.2	— 2.4	— 4.8	— 9.6	— 19.2	— 1.8	— 3.6	— 7.2	— 14.4	— 28.8

It is evident that multiplying both K_1 and L_1 by any given number also multiplies α and G_1 by that number. The two halves of each table have been computed for values of K and L such that in each succeeding column their values are twice those of the preceding column, hence the values of α (and β) follow the same law. So that after one column of either table has

been computed the others are simple multiples of it. Additional columns may readily be made. The coordinates given in these tables suffice for plotting many loci. Of these a few have been selected for consideration. $G_2 = 2$ has been assumed for all of the loci in Fig. 22 and more or less complete loci have been plotted for values of $G_1 = \frac{1}{4}, \frac{1}{2}, 1, 2, 4$, and 8.

Numbers placed at various points along these loci designate the values of ω at those points.

Let us continue the consideration of the locus $G_1 = \frac{1}{2}$, $G = 2$. It is very nearly circular until near P, ($\omega = 15$) *i. e.* at frequencies below that the current through the branch 2 is comparatively small but after that, I_2 rapidly assumes a much greater relative importance and so the locus makes a wide detour from its previous course until at about $\omega = 26$, $\alpha = \beta$, and both components having the same lag the locus reaches the circumference of the circle of junction $O\ Q_1$ becomes tangent to it and passes into the other leaf. It continues near the circumference $O\ Q$ until ω becomes infinite at 0.

The frequency at which the locus becomes tangent to $O\ Q$ is to be found from the equation

$$\alpha = \beta, \text{ i. e., } \frac{K_1 - L_1 \omega_0^2}{R_1} = \frac{K_2 - L_2 \omega_0^2}{R_2}$$

$$\therefore \omega_0^2 = \frac{\frac{K_1}{R_1} - \frac{K_2}{R_2}}{\frac{L_1}{R_1} - \frac{L_2}{R_2}} \tag{30}$$

From (30) we find for this locus $\omega_0^2 = 700$, $\therefore \omega_0 = 26.45$. Since frequency must be positive it appears that a given locus never has more than one point of tangency with the circle $O\ Q$ other than 0. Following is a table of values of ω_0^2 for various relative values of G_1 and G_2 for the two branches under consideration. It is evident that multiplying K_1, K_2, L_1 and L_2 in (30) by any given number, as m will multiply both G_1 and G_2 by that number but will leave ω_0^2 unchanged. This finds expression in the table in the fact that the values of ω_0^2 are the same in any diagonal sloping downward to the right.

Values of ω_0^2 in equation (30) for $R_1 = 5$ and $R_2 = 13$.

$G_1 =$	m	$2m$	$4m$	$8m$	$16m$	$32m$	$64m$	$128m$	$256m$	$512m$
$G_2 = m$	-200	0	57	80	90.3	95.2	97.6	98.8	99.4	99.7
$2m$	$\mp\infty$	-200	0	57	80	90.3	95.2	97.6	98.8	99.4
$4m$	700	$\mp\infty$	-200	0	57	80	90.3	95.2	97.6	98.8
$8m$	500	700	$\mp\infty$	-200	0	57	80	90.3	95.2	97.6
$16m$	443	500	700	$\mp\infty$	-200	0	57	80	90.3	95.2
$32m$	420	443	500	700	$\mp\infty$	-200	0	57	80	90.3
$64m$	410	420	443	500	700	$\mp\infty$	-200	0	57	80
$128m$	405	410	420	443	500	700	$\mp\infty$	-200	0	57
$256m$	402	405	410	420	443	500	700	$\mp\infty$	-200	0
$512m$	401	402	405	410	420	443	500	700	$\mp\infty$	-200

The value of ω_0^2 for the principal diagonal of the table may be expressed more simply, since then

$$G_1^2 = G_2^2 \text{ or } \frac{K_1 L_1}{R_1^2} = \frac{K_2 L_2}{R_2^2}$$

$\therefore$ by comparison of proportions,

$$\frac{\frac{K_1}{R_1} \quad \frac{K_2}{R_2}}{\frac{L_2}{R_2} \quad \frac{L_1}{R_1}} = -\frac{\frac{K_1}{R_1} - \frac{K_2}{R_2}}{\frac{L_1}{R_1} - \frac{L_2}{R_2}} = -\omega_0^2 \qquad (31)$$

Let ω_{01}^2 and ω_{02}^2 be values of ω_0^2 on two diagonals at equal distances from the principal diagonal of the table and let K_1, L_1, K_2, L_2, designate the values these quantities have in the first square of the diagonal then their values in the nth square of the top line are $2^n K_1$, $2^n L_1$, K_2, L_2, and in the nth square of the first column K_1, L_1, $2^n K_2$, $2^n L_2$; hence

$$\omega_{01}^2 = \frac{\frac{2^n K_1}{R_1} - \frac{K_2}{R_2}}{\frac{2^n L_1}{R_1} - \frac{L_2}{R_2}} \qquad \omega_{02}^2 = \frac{\frac{K_1}{R_1} - \frac{2^n K_2}{R_2}}{\frac{L_1}{R_1} - \frac{2^n K_2}{R_2}}$$

$$\therefore \omega_{01}^2 \, \omega_{02}^2 = \frac{2^n \left(\frac{K_1}{R_1} - \frac{K_2}{R_2}\right) + (2^n - 1)\frac{K_1 K_2}{R_1 R_2}}{2 \left(\frac{L_1}{R_1} - \frac{L_2}{R_2}\right) + (2^n - 1)\frac{L_1 L_2}{R_1 R_2}}$$

Hence by composition of proportions in equation (31) we have

$$\omega_{01}^2 \, \omega_{02}^2 = \omega_0^4$$

i. e. the product of two tabular numbers equidistant from the principal diagonal is equal to the square of a number in that diagonal.

Furthermore the tabular numbers increase above that diagonal and decrease below it, to definite limits; for, considering numbers near the right hand end of the upper line in the table as computed by eq. (30), it will be noticed that the first terms of the numerator and denominator are so large compared with the last terms that the value of ω_0^2 approaches the limit K_1 / L_1, obtained by neglecting the last terms. That limit in the present example is 100. But if numbers near the bottom of the first column of the table be considered, the last terms of the numerator and denominator of eq. (30) are large and the limit in this case is K_2 / L_2, or 400 in the present example.

For frequencies between the limiting values $\omega_0^2 = K_1 L_2$ and $\omega_0^2 = K_2 / L_2$ these loci have no points of tangency on $O\ Q$. In our example no locus is tangent to $O\ Q$ between the frequencies $\omega = 10$ and $\omega = 20$.

It will be noticed also that in this table ω_0^2 is negative and ω_0 is consequently imaginary in case $G_1 = G_2$ and generally in this table unless either $G_1 \gtrless 2\ G_2$ or $G_2 \gtrless 2\ G_1$, *i. e.*, loci lying between these relative values of G_1 and G_2 have no points of tangency with $O\ Q$.

It is then evident that certain of the loci have points of tangency with $O\ Q$ outside definite limits, while others have no such points of tangency anywhere.

Again, let $F = \tan \theta_1 / \tan \theta_2$

$$\therefore F = \frac{(K_1 - L_1 \omega^2)\ R_2}{(K_2 - L_2 \omega^2)\ R_1} = \frac{(K_1 / \omega^2 - L_1)\ R_2}{(K_2 / \omega^2 - L_2)\ R_1} \qquad (32)$$

It is evident that for small values of ω the value of F is approximately

$$F' = \frac{K_1}{R_1} \div \frac{K_2}{R_2} \qquad (33)$$

hence any locus in which ω alone is variable approximates near O (where ω is small) to the locus represented by equation (33), a locus which has been fully treated in connection with Fig. 13 that part of any locus in which ω alone varies, which is near O, lies therefore between the upper semi-circles on the diameters $O\ Q$ and $O\ E_2$.

Likewise for large values of ω, in case ω alone varies, the locus approximates to that represented by the equation

$$F'' = \frac{L_1}{R_1} \Big/ \frac{L_2}{R_2} \tag{34}$$

which near O lies between the lower semi-circles on $O\,Q$ and $O\,E_2$. Whether the locus starts out from O in the first leaf or not, depends upon the value of F''. In case $F' > 1$, *i. e.*,

$$\frac{K_1}{R_1} > \frac{K_2}{R_2}$$

then the first portion of the locus, where ω is small, lies in the first leaf, but in case $F' < 1$ it lies in the second leaf, as was shown in connection with Figs. 19 and 20.

Again, in case $F'' > 1$, *i. e.*,

$$\frac{L_1}{R_1} > \frac{L_2}{R_2}$$

the locus terminates at O (for large values of ω) in such a manner that the last portion of it lies in the second leaf; but in case $F'' < 1$ it lies in the first leaf. In one case of $G_1 = \frac{1}{3}$ and $G_2 = 2$, $F' = \frac{1}{3}$ and $F'' = \frac{1}{3}$. This locus, therefore, starts in the second leaf at $\omega = O$, and passes from the second to the first leaf at $\omega_0 = 26.45$ and terminates at $\omega = \infty$ in the first leaf.

The most remarkable and important property of the locus $G_1 = \frac{1}{3}$, $G_2 = 2$ for practical applications is this: for frequencies in the wide interval from $\omega = 10$ to $\omega = 20$ the working current varies only a few per cent. with a minimum near $\omega = 15$ and maxima near 11 and 19. The total effect then of this divided circuit as compared with the single circuit $R_1\,K_1\,L_1$ is to permit in the interval $10 < \omega < 20$, a somewhat greater working current to flow than $I = O\,E_1$ which would have flowed in the first branch at $\omega = 10$, but the current is so modified as to have a stable minimum at $\omega = 15$ instead of the unstable maximum at $\omega = 10$ in the single circuit. The possible modifications which can be introduced by simple branching suitably designed to effect automatic regulation of various kinds seems to the author to be a field in alternating current engineering likely to be of great importance to the inventor. Such a device introduced for instance into an alternating circuit would quite do away with the necessity for any very close regulation of speed by the governor.

Next consider the locus in case $G_2 = 2$ and $G_1 = 2$ as shown in Fig. 22. The locus starts in the second leaf, for by equation (33) $F < 1$; but it has no point of tangency with the circle on $O\ Q$ for $\omega_0^2 < 0$ and ω_0 is imaginary.

By equation (34) $F'' > 1$, hence it also terminates at O in the second leaf.

But besides this it has points of contact with the inner edge of the leaf on the circumference $E_1\ E_2$. As shown in connection with Figs. 17 and 18, the numerical value of $\alpha\ \beta = G^2 = 1$, at all points of this circumference an $E_1\ E_2$. Hence to find such points of tangency we have the equations

$$\frac{(K_1 - L_1\ \omega^2)\ (K_2 - L_2\ \omega^2}{R_1\ R_2\ \omega^2} = -1,$$

or

$$\tan\ \theta_1\ \tan\ \theta_2 = -1$$

or

$$\theta_2 - \theta_1 = \pm\ 90°$$

$$\therefore\ \omega^2 = \tfrac{1}{2}\left[\frac{K_1}{L_1} + \frac{K_2}{L_2} - \frac{R_1\ R_2}{L_1\ L_2} \pm \sqrt{\left(\frac{K_1}{L_1} + \frac{K_2}{L_2} - \frac{R_1\ R_2}{L_1\ L_2}\right)^2 - 4\frac{K_1\ K_2}{L_1\ L_2}}\right] \tag{35}$$

Designate these roots as ω_1^2 and ω_2^2; their values are given in the accompanping tables in which imaginary values are designated by i.

Values of ω_1^2 and ω_2^2, equation (35).

$G_1 =$	0.25	0.5	1	2	4	8
$G_2 = 0.25$	i	i	i	i	i	200 200
0.5	i	i	i	i	200 200	122 328
1	i	i	i	200 200	122 328	110 364
2	i	i	200 200	122 328	110 364	104.5 383
4	i	200 200	122 328	110 364	104.5 383	102.1 391.6
8	200 200	122 328	110 364	104.5 383	102.1 391.6	101.1 395.8

$G_1 =$	0.375	0.75	1.5	3	6	12
$G_2 = .375$	i	i	i	i	138 ·53	119 337
0.75	i	i	i	158 ·53	119 317	108 370
1.5	i	i	158 ·53	119 337	108 370	104 385
3	i	158 ·53	119 337	108 370	104 385	102 392
6	158 ·53	119 337	108 370	104 385	102 392	101 396
12	119 337	108 370	104 385	102 392	101 396	100.5 398.

Since

$$\omega_1^2 \, \omega_2^2 = \frac{K_1 \, K_2}{L_1 \, L_2}$$

we have $\omega_1^2 \, \omega_2^2 = 40{,}000$ throughout this table. All values of ω_1 and ω_2 lie between 10 and 20, and whenever ω_1^2 is real ω_2^2 is also real, or in other words, whenever a locus has one point of tangency with the inner edge, it also has a second such point, though it is possible for the two points to come together, in which case the locus will have a point of double contact with the inner edge.

On the locus $G_1 = G_2 = 2$, $\omega_1 = 11_1 +$ and $\omega_2 = 18. +$. At the first of these points the locus passes from the second leaf to the first, and at the other point it returns to the second leaf, on which it remains for all values of ω greater than 18. $+$.

Consider also the locus $G_2 = 2$. $G_1 = 8$. In this case $F' > 1$ and $F'' > 1$. Hence the locus begins in the first leaf and ends in the second. It passes at

$$\omega_0 = \sqrt{57. +} = 7.5$$

from the first leaf to the second. It also intersects both the previous loci and all other loci for which $G_2 = 2$ at $\omega = 10$ and passes again upon the first leaf from

$$\omega_1 = \sqrt{104.5} = 10.2 + \text{ to } \omega_2 = \sqrt{388} = 19.6 +.$$

The striking peculiarity of both these loci is a strongly marked minimum working current, such that a far greater power must be exerted to make the frequency much greater or less than that corresponding to this minimum. Such a split current constitutes an effective automatic speed regulator.

Part of the locus $G_2 = 2$, $G_1 = 1$ has been constructed. It is interesting from the fact that $\omega_3^2 = \pm \infty$ and

$$\omega_1 = \omega_2 = \sqrt{200} = 14. +.$$

It therefore has a double contact with the outer edge at O and with the inner edge $\omega = 14. +$.

Part of the locus $G_2 = 2$, $G_1 = \frac{1}{4}$ is also shown. The portions of these last loci not drawn above the upper semi-circle $O\,E_1$, lie intermediate between those already plotted.

Plotting loci for successive values of G_1 is facilitated by drawing auxiliary loci for successive constant values of ω. These auxiliary loci are circles like those in Figs. 15 and 16, as is evident from equations (25) and (26.) For assuming K_2, L_2, R_2 and ω as given, G_2 and β are known constants, but with K_1 and L_1 variable, G_1 and α are also variable. As shown in connection with Figs. 15 and 16, the auxiliary loci for β constant and α variable are circles whose centers lie on the circle of diameter $C_1\,C_1'$. Parts of these circles have been drawn on Fig. 22 for values of $\omega = 15, 20, 25, 30$.

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